Discrete Choice Models of Firms' Strategic Decisions

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1 Introduction

Many key strategic decisions firms make involve discrete choices such as deciding the location of a new store, determining where in product space to position a product, or what options to offer in a service contract. These decisions are fairly complex and typically involve the consideration of a number of demand, cost, and competitive factors. However, researchers frequently only have access to data revealing the final choice the firm made. What makes these discrete choices particularly interesting (and challenging to analyze) is that they are interrelated with the choices of other firms.

The interrelation in choice behavior stems from the fact that firms take into account the actions of their competitors when making their own decisions. For example, firms are influenced in their choice of location by their expectations of where their competitors will locate (and vice versa). Similarly, when choosing what kind of pricing strategy to adopt (EDLP vs. Hi-Lo) firms base their decisions on what they expect their competitors will do. Thus, modeling firms' discrete decisions requires describing firms' beliefs and equilibrium behavior. While marketing researchers are familiar with the use of revealed preference arguments and latent utility models to estimate consumer preference functions, less is known about how to adapt these models to study firms' discrete choices when they are interrelated.[1] One of our goals is to illustrate how a game-theoretic framework can aid in the construction and estimation of interrelated choice models.

The study of interrelated discrete decisions poses several methodological challenges such as large state spaces, the presence of multiple equilibria and dynamics in firm decisions. Fortunately, there now is a growing literature on how to resolve some of these issues in static games (see the recent survey by Berry & Reiss 2007). More recently there has been interest in tackling dynamic issues (see the survey by Doraszelski & Pakes 2007) and novel estimation methods have been developed to deal with them (Pesendorfer & Schmidt-Dengler 2003, Aguirregabiria & Mira 2007, Bajari, Benkard & Levin 2007, Berry & Pakes 2007).

The availability of methods to estimate interrelated discrete choice models opens up new avenues for applied empirical research in marketing and economics. Examples include examining supermarket pricing strategy (Ellickson & Misra 2007b),

[1] For an excellent survey of the growing literature in social interactions in a consumer context, see Hartmann et al., "Interdependent Choices and Social Multipliers: Identification, Methods and Policy Implications" in this issue.

retail location decisions (e.g. Seim 2006, Zhu & Singh 2006), product offerings across markets (Draganska, Mazzeo & Seim 2007), technology diffusion and network effects (Ryan & Tucker 2007) and stock buy-sell recommendations (Bajari, Hong, Krainer & Nekipelov 2006). While these new methods have encouraged applied work in marketing and economics on new and interesting topics, there remains a number of open questions.

In this article we first describe the basic problem of dealing with interrelated discrete choices in a game-theoretic framework and outline the various estimation methods available and the challenges they present. Then we turn our attention to a discussion of the existing applications and identify future research opportunities.

2 Modeling Framework

We present a simple example of a latent variable model of interrelated decisions to fix ideas. Consider the two player framework described by Bresnahan & Reiss (1991b). Each player (firm) chooses one of two possible actions (to enter a market or not) based on expected profits. It is assumed that these outcomes are the result of a static game. In this normal-form game the payoff matrix for player 1 is

	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	Π^1_{00}	Π^1_{00}
$a_1 = 1$	$\Pi^1_{00} + \Delta^1_{10}$	$\Pi^1_{00} + \Delta^1_{10} + \Delta^1_{11}$

In the above table, Π^1_{00} denotes the profits that accrue to player 1 if she chooses not to enter, i.e. if $a_1 = 0$. On the other hand, if player 1 chooses to enter, $a_1 = 1$ and her profits will depend on the action chosen by player 2. Simple algebra shows that player 1 will choose to enter if

$$\Delta^1_{10} + a_2\Delta^1_{11} \geq 0. \tag{1}$$

In other words, player 1 will choose that action that gives her maximum profits. Taking the above expression to data seems straightforward by adding an error component until one realizes that the inequality described above also contains player 2's decision.

Let player 2's payoff matrix be

	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	Π^2_{00}	$\Pi^2_{00} + \Delta^2_{01}$
$a_1 = 1$	Π^2_{00}	$\Pi^2_{00} + \Delta^2_{01} + \Delta^2_{11}$

Using the same logic as for player 1, an analogous expression for player 2's entry decision can be written as,

$$\Delta^2_{01} + a_1 \Delta^2_{11} \geq 0. \tag{2}$$

The two equations together can be used to outline a system of discrete choice equations that must be estimated jointly. A simple way to achieve this is to simply make the Δ's a function of observed covariates,

$$\Delta^1_{10} = X'\beta_{11}, \Delta^1_{11} = X'\beta_{12}, \Delta^2_{01} = X'\beta_{21}, \Delta^2_{11} = X'\beta_{22}. \tag{3}$$

and then to add errors. In this way we obtain the following inequalities, which can be taken to the data.

$$\begin{aligned} X'\beta_{11} + a_2 X'\beta_{12} + \epsilon_1 &\geq 0. \\ X'\beta_{21} + a_1 X'\beta_{22} + \epsilon_2 &\geq 0. \end{aligned} \tag{4}$$

The goal of interrelated discrete choice models such as the one presented above is to recover information about consumer preferences or firm profits including how other agents' decisions influence a given agent's payoffs. Although interactions between economic agents occur in a variety of settings and contexts there is a common set of modeling choices that affect the econometric specification, identification and estimation. Broadly, these choices can be classified along three dimensions:

- *The Informational Context: Complete vs. Private Information*
- *The Temporal Context: Static vs. Dynamic Games*
- *The Timing of Moves: Games with Simultaneous vs. Sequential Moves*

In general, the details of the application will dictate what assumptions are made and therefore the type of discrete choice model implemented. Below we illustrate the implications these assumptions have on the structure of the discrete choice model.

Informational Context: Complete vs. Private Information. Bresnahan & Reiss (1991b) assume this game to be one of *complete information*, that is, each player knows the action of its competitors. In this case the ϵ's are known to the firms but not to the researcher. This case parallels the errors that appear in standard discrete choice models. An alternative is to assume that the errors constitute *private information* for each player. In this case, each firm knows its own ϵ but not the ϵ's of the other firms and each player must form beliefs about the choices of the other players. Two key assumptions are typically added at this point: (a) the researcher does not know any of the ϵ's and (b) the researcher's uncertainty has the same distribution as that of the firms' (symmetric) uncertainties. The researcher posits the rational beliefs of players, namely, that in equilibrium the beliefs about players' likelihoods of entry are consistent and writes

$$\begin{aligned} P(a_1 &= 1) = P(X'\beta_{11} + P(a_2 = 1)X'\beta_{12} + \epsilon_1 \geq 0), \\ P(a_2 &= 1) = P(X'\beta_{21} + P(a_1 = 1)X'\beta_{22} + \epsilon_2 \geq 0). \end{aligned} \tag{5}$$

Notice that in the above equations the indicators of competitor choices have been replaced by their expectations (the probabilities of such choices). This system of equations then yields a fixed point for the probabilities, which constitutes the equilibrium. The likelihood to be maximized is,

$$L = \prod_{i \in 1,2} P(a_i = 1)^{Y_i} \left[1 - P(a_i = 1)\right]^{1-Y_i}.$$

Since the model implies a probability distribution over the possible outcomes, a natural starting point is to construct a nested maximum-likelihood algorithm that, in each iteration, solves the fixed-point problem given the current estimate of the parameter values. We discuss estimation in more detail in a later section.

Temporal Context: Static vs. Dynamic Games. The above model presumes static interactions: all actions are taken in the same period and payoffs are realized as soon as such actions are taken. Clearly, this is a very restrictive framework. Most researchers will agree that strategic interactions between firms occur over time. This requires us to model firms as forward looking entities that maximize some form of discounted profits. Keep in mind that firms now also have to either know the choices that their competition will make in the future or have to form conjectures about such actions.

In dynamic games, firms are assumed to maximize the net discounted value of the payoffs from choosing a particular sequence of actions taking as given their expectations about competitor actions. This program can be written as

$$W_i\left(s,\epsilon_i;P\right)=\max_{a_i\in A_i}\left\{\begin{array}{c}\sum\limits_{a_{-i}}\Pi_i\left(a_i,a_{-i},s\right)P_{-i}\left(a_{-i}|s\right)+\epsilon_i\left(a_i\right)\\ +\beta\int\sum\limits_{a_{-i}}W_i\left(s',\epsilon_i';P\right)g\left(s'|s,a\right)P_{-i}\left(a_{-i}|s\right)f\left(\epsilon_i'\right)d\epsilon_i'ds'\end{array}\right\} \quad (6)$$

where s is a vector of state variables, a_i is the action of player i, P_i is the probability of choosing action a_i (correspondingly for competitor firms we have P_{-i} and a_{-i}). Finally, $\Pi_i\left(a_i,a_{-i},s\right)$ is the per period profits that are conditioned on actions and the state vector. Given these constructs, the ex ante value function[2] can be written as

$$V\left(s\right)=\int W_i\left(s',\epsilon_i';P\right)f\left(\epsilon_i'\right)d\epsilon_i'. \quad (7)$$

Firm i then chooses the action (a_i) that maximizes

$$V\left(a_i,s\right)=\sum_{a_{-i}}\Pi_i\left(a_i,a_{-i},s\right)P_{-i}\left(a_{-i}|s\right)+\beta E\left[V\left(s'\right)|s,a_i\right]+\epsilon_i \quad (8)$$

The econometric implementation of such dynamic games is challenging, to say the least. Recent research (Aguirregabiria & Mira 2007, Bajari, Benkard & Levin 2007) has made it easier to estimate such dynamic games, albeit under very particular assumptions. While dynamic games are more realistic, static games are usually econometrically less complicated to estimate. It is an open question how well static games approximate the steady-state outcomes of dynamic games.

Timing of Moves: Games with Simultaneous vs. Sequential Moves. The model in Section 2 also presumes that players act simultaneously. This assumption of course has direct implications for the interpretation of the strategic interactions and the estimation of the game. For example, if we have a two-player game where one player moves first then the two players have different information sets which have to be modeled by the econometrician. In other words, the first mover has to form

[2] Also known as the social surplus function.

conjectures about the reaction of the other player but not vice versa. This asymmetry has to be incorporated in the estimation framework.

As an illustration, consider the case where the order of the moves (o) is not observed and let $\mathcal{N}_o$ be a set containing all possible permutations of actions. The probability of observing a given set of actions can then be depicted as,

$$P(a) = \sum_{o \in \mathcal{N}_o} P(a|o) P(o). \tag{9}$$

More generally, if players move sequentially and the sequence of moves is observed, then that information can be used to inform the parameters because for a given order of moves, the equilibrium is unique. (see e.g. Bresnahan & Reiss 1991a). If the sequence of moves is not observed then the problem is more pronounced since the researcher has to integrate over all possible sequence combinations (see e.g. Einav 2003). However, even if the exact order of moves is unobserved, identification can become easier if there exist variables that affect the timing of the moves but not the payoffs. If such variables (say z) are available they can be incorporated straightforwardly as follows,

$$P(a) = \sum_{o \in \mathcal{N}_o} P(a|o) P(o|z). \tag{10}$$

These move-shifting variables act as exclusion restrictions that help separate the order of the moves from the estimation of the payoff function effects.

3 Estimation Issues

The interrelatedness of firm decisions and the game-theoretic nature of the framework give rise to a number of issues and require modifications to the standard discrete choice estimation methodology. We discuss these below.

Estimation Methods. To illustrate the estimation problem consider the estimation of the model presented in Section 2. Under the assumption of private information we would have to construct a likelihood based on equation (5). To illustrate the estimation approaches consider a simpler case: Define, for $i = 1, 2$,

$$\Psi(a_i = 1) = \frac{\exp(\alpha + X'\beta + \delta \times \Pr(a_{-i} = 1))}{1 + \exp(\alpha + X'\beta + \delta \times \Pr(a_{-i} = 1))}. \tag{11}$$

The likelihood is then defined by

$$\mathbb{L} = \prod_{j \in 1,2} \Psi\left(\sigma_j = 1\right)^{Y_i} \left(1 - \Psi\left(a_i = 1\right)\right)^{1-Y_i} \tag{12}$$

subject to the equilibrium mapping constraints

$$\Psi\left(a_i = 1\right) = \Pr\left(a_i = 1\right). \tag{13}$$

The key econometric problem stems from the fact that for any guess of the parameters there is a fixed point (possibly more than one) in the probability space as seen by the equilibrium constraints. In other words, at every iteration of the optimization procedure the evaluation of the likelihood requires the researcher to solve for such fixed points. In addition, if there is more than one fixed point (equilibrium), the researcher has to also prescribe an equilibrium selection rule. While this full information maximum likelihood approach (commonly referred to as nested fixed point method) has been used in the estimation of single agent dynamic discrete choice models (Rust 1994) most applications to games are in the context of static games (see e.g., Seim 2006, Orhun 2006).

The computational cost of the nested fixed point algorithm has prompted the development of alternative methods such as the two-step approach of Bajari, Benkard & Levin (2007) and the nested pseudo likelihood by Aguirregabiria & Mira (2007). The two step approach avoids the computation of the fixed point by first estimating the beliefs that players have about their competitors' actions (i.e., consistently obtaining $\widehat{\Pr}\left(a_{-i} = 1\right)$ and then plugging them into a structural second stage model (defined by (11)) . The basic idea here is relatively simple. Since the fixed point in probability space (i.e. the equilibrium) is only a function of the observable state variables, a first stage that models firms' choices only as a function of these variables (ignoring the strategic effects) using flexible methods is sufficient to obtain consistent estimates of the probabilities. These first-stage probabilities are, in essence, estimates of the beliefs players have about competitive actions. The recovered beliefs can then be plugged into a second stage model that also accounts for competitive effects. Of course, one should be careful to have exclusion restrictions in the first stage that allow these effects to be identified.

In the case of nested pseudo likelihood methods the process continues iteratively.

That is, we do not stop at the second stage but use the resultant probabilities $\left(\widehat{\Psi}\left(a_i=1\right)\right)$ as our new (and arguably more efficient) estimates of firms' beliefs (i.e. now we set $\Pr\left(a_i=1\right)=\widehat{\Psi}\left(a_i=1\right)$). This iterative process continues until convergence. The underlying assumption of both approaches is that there is only one equilibrium being played in the observed data.

To deal with the computational complexity of likelihood estimation without the necessity of strong assumptions, Judd & Su (2007) advanced a direct optimization approach called MPEC (Mathematical Programming with Equilibrium Constraints). The key idea is to choose parameters and endogenous variables to maximize the likelihood subject to the constraint that the endogenous variables are consistent with the equilibrium determined by the structural parameters. A recent application to a discrete game of incomplete information is Vitorino's (2007) study of the shopping center industry.

An alternative to the likelihood-based approaches are method of moment (GMM) estimators. These estimators rely on the specification of moment conditions based on a structural game. The moment conditions are then used to construct an objective function that is minimized (Thomadsen 2005, Draganska et al. 2007). There is little specific guidance as to how to decide which moment conditions to use. The general consensus seems to be to match all the moments that are valid given the model and the assumptions, and if there are more moments than parameters, to apply the efficient weight matrix and test for the validity of the overidentifying restrictions.

Multiplicity of Equilibria. A key problem with discrete choice models with strategic interactions is the presence of multiple equilibria. It is useful to distinguish two different problems: multiple equilibria in the model and multiple equilibria in the data.

Multiplicity in the model arises on account of the inherent nonlinearity of the response functions. This problem manifests itself in the form of the existence of more than one equilibrium for any given parameter set. Even if one could search and find all such fixed points, their existence then implies that an equilibrium selection mechanism be assumed in order to construct the likelihood. Questions emerge about the types of selection criteria a researcher could and should adopt (Berry & Reiss 2007). Most approaches, such as the two-step approach proposed by Bajari, Benkard & Levin (2007) avoid the multiplicity problem in estimation by assuming that there is only one equilibrium played in the observed data. The MPEC method (Judd & Su 2007) discussed above presents an alternative when this is not the case.

In the literature of empirical discrete games, as far as we know, only two papers have dealt with the problem of multiple equilibria in the data. Sweeting (2005) shows how one can use the multiplicity of equilibria as an identification device. While his approach works well in a binary setting, it remains to be seen if the idea can be extended to more complex choice situations. Bajari, Hong & Ryan (2007) adopt a more data-driven approach by allowing for a probabilistic mix of alternative equilibrium selection rules and allowing covariates to influence the choice of such rules.

Common Unobservables. Most estimators of discrete games assume that any unobservable components in the agents payoffs represent private information and thus these unobservables are treated as i.i.d. errors. At the same time, most applied researchers agree that there is often information that is common knowledge to the players but unobserved by the researcher. If the players have common information about the discrete choices (e.g.. location, product), then ignoring such common unobservables will create a simultaneity bias in estimating the effect of strategic interactions among these players on their payoffs from the revealed choice data. For example, firms may all choose to enter a given market because they know of a favorable (but unobserved to the researcher) demand condition. Ignoring this common information may make it seem like firms are coordinating actions. Orhun (2006) controls for location-specific common unobservables when estimating the location choice of supermarkets and demonstrates that ignoring common information that changes the probability of all players in opting for a particular discrete choice results in a bias in the estimate of the tradeoff between the strategic effect of other players and the attractiveness of a choice measured by observables.

Dealing with common unobservables in two-step estimators is difficult. In general, we do not have methods that provide consistent nonparametric estimates of choice probabilities conditional on common unobservables. In other words we can no longer recover the beliefs of players from the data in the first stage. For this reason, most applications with common unobservables have adopted either the Nested Fixed Point (Seim 2006, Zhu & Singh 2006) or the Nested Pseudo Likelihood approach (Ellickson & Misra 2007b, Aguirregabiria & Mira 2007, Collard-Wexler 2005). We should note that while both these approaches offer ways to deal with common unobservables, the treatment of such effects remains limited and is the focus of continuing work.

A related issue is that of multimarket contact and joint decision making across multiple markets. Assuming i.i.d. errors in these cases does not seem like a plausible assumption and alternative ideas need to be articulated which might help in dealing with this problems. A first step in this direction is the work of Jia (2006), who considers the interdependence of firms' decisions across locations.

Statistics and Inference. Estimators typically require some statement about standard errors or confidence regions to effectively communicate the significance of the estimated effect. In the type of models discussed here standard estimators of parameter covariances such as those obtained by inverting the observed information matrix or approximations thereof do not necessarily apply on account of the pseudo-likelihood nature of the objective function. Simple bootstrap and jackknife estimators are better candidates but are also not straightforward to implement. The key question here is what the the relevant "units" are. Clearly, one cannot bootstrap over players since that would change the structure of the game. Similarly, in a dynamic model one cannot simply bootstrap over time. The ideal scenario is one where the researcher has data across a number of markets and the bootstrap is implemented using markets as observations (Ellickson & Misra 2007b). Other methods such as subsampling also offer alternatives to classical bootstrap and jacknife methods and might be better suited to the estimators discussed here.

A related but equally important issue is that of approximation errors in dynamic games. The simulation and interpolation methods which are used to approximate expected discounted values (as well as the nonparametric methods which are used to estimate of players' choice probabilities) provide only approximations to the solution of dynamic games or to the expected value of an strategy function. We know very little on the effects of these approximation errors on the properties of our estimators, except that these estimators are biased and that the bias is potentially important. There remains a lot to learn in this area.

Counterfactuals and Policy Experiments. As mentioned earlier, two-step approaches such as the one proposed by Bajari, Benkard & Levin (2007) avoid the multiplicity problem in estimation but any policy experiments/counterfactuals nevertheless requires the solution and selection of such fixed points.

In a recent paper Aguirregabiria & Ho (2006) offer a simplifying approach. They posit that in games of incomplete information, it is possible based on the continuous differentiability of the equilibrium mapping (both with respect to the structural parameters and with respect to the vector of choice probabilities) to use a Newton method to obtain a linear approximation to the counterfactual choice probabilities. As long as the equilibrium selection mechanism is a smooth function of the structural parameters, this approximation is valid for any equilibrium selection mechanism. In general, ideas and methods that work around this problem will be of value in applications where even solving for one such fixed point might involve a large computation burden.

An additional related issue is that of constructing structural models that describe post-entry firm interactions in the market. The earliest work in this area is due to Reiss & Spiller (1989). They adopt a structural framework in the context of airline markets that allows for the inclusion of price and quantity data in addition to the entry decision information. The problem with constructing such structural models and including post-entry outcome data is that there are severe selectivity (i.e., we only observe prices and quantities if a player enters) and endogeneity (i.e., players actions are correlated to common unobservables) problems. There is some recent work (Draganska et al. 2007, Ellickson & Misra 2007a) that attempts to resolve these issues and extend the literature in this direction.

Discussion

The models and methods outlined in this paper require an added degree of theoretical and econometric effort. There is some natural skepticism in the marketing (and economics) community regarding the increased complexity these model require and the value generated. In what follows we outline three key reasons why incorporating strategic effects is critical.

First, ignoring strategic effects can bias the estimates of parameters included in the payoff function. Examples of such bias are documented, for example, in Zhu & Singh (2006) and Ellickson & Misra (2007b). The underlying reason for the bias is straightforward: If strategic effects matter and are ignored then the other factors included in the payoffs will be estimated with a bias. The magnitude of such bias will depend on the degree to which strategic effects matter. In some cases these can be severe, and even go as far as changing the sign of the effects of interest.

A second, more conceptual, problem is that without the incorporation of strategic effects counterfactuals and policy experiments can be difficult. In other words, while reduced form models might be able to give us some coarse insights into the effects of market and firms characteristics on decisions they will be unable to go much beyond that. Draganska et al. (2007) demonstrate how the insights obtained from merger analysis dramatically change once firms are allowed to react strategically and optimally adjust their product assortments.

Finally, the incorporation of strategic effects often allows for the inclusion of competitor characteristics (via excluded variables and fixed effects) into the payoffs of a given firm. While the main purpose of such exclusions is to aid identification, they also have a side benefit of improving the fit to the data. Without the strategic effects, there would be no particular reasons why the characteristics of a firm's competitors would enter the payoff function of a firm. Moreover, the predictions from a model without strategic effects would be weaker. Ellickson & Misra (2007b) find that strategic effects explain a large proportion of the variation in strategy choices in supermarket pricing. In summary, even though the computational burden imposed by these models is high, the costs imposed are more than offset by the benefits that accrue.

4 Applications

Marketers and economists are inherently interested in how firms make decisions, especially in competitive environments. The methods described in the earlier sections allow the researcher to answer numerous questions in this broad area with fairly limited data. Examples of such questions include

- How do market factors influence profits?
- How does a firm react to competition?
- Are firms choosing certain strategies as a differentiation device or to coordinate?
- Are firms forward looking? How does this impact their decisions and the decisions of competing firms?

While these are fairly general questions, they take on added importance when embedded in the context of a particular industry or market. For example, we can address specific questions such as: How does the expected entry of Wal-Mart affect the profits of other discount stores? Why do we see some supermarkets offer sales and others choosing an everyday low price policy? Why do some ice-cream brands offer a variety of flavors while others do not? In what follows we will highlight some of the work done so far and provide directions for future applications. We start with a discussion of the entry literature and then move to applications related to the marketing-mix decisions (4 P's).

Entry decisions/Timing of entry. The empirical literature on firm entry decisions in oligopolistic markets started with Bresnahan & Reiss (1987) and Bresnahan & Reiss (1991a) who modeled a situation where firms' revenues and costs are not observed, but their entry decisions are. They show how the number of firms varies with changes in demand and the degree of competition. The empirical application is limited to industries and occupations in which all sellers of a narrowly defined product can be identified: doctors, dentists, druggists, plumbers, and tire dealers. Another early paper on firm entry was Berry (1992), which looked at entry of airlines into specific city-pair markets. Ciliberto & Tamer (2006) expand on these earlier entry papers by considering a two-stage model of entry, where in the first stage firms decide whether or not to enter and in the second they compete in quantities. Their fairly flexible model has multiple equilibria, so they give up on point identification and identify the bounds of the entry probabilities.

Another recent direction taken by the literature on entry has focused on the inherent dynamics in these decisions. Collard-Wexler (2005) investigates the role of demand fluctuations in determining plant entry and exit decisions in the ready-mix concrete industry. The early entry literature, by focusing on one snapshot of data, also results in estimates that necessarily reflect market conditions at the time. The fundamental assumption underlying these models is that they apply to industries that have reached a stable, long-term equilibrium. Recent work by Dunne, Klimek, Roberts & Xu (2006) and Orazem & Xiao (2006) consider dynamic extensions of the basic Bresnahan and Reiss model to investigate whether entry conditions differ over time, violating the assumption of earlier work.

Place and distribution strategy. Once the entry decision has been made, firms need to decide on the precise position in the market they have entered. Particularly in retail, location is one of the most important strategic decisions and has been the main focus of empirical work in this area. Modeling firm locations involves a significant methodological challenge due to the size of the problem. In making their location choice, firms have to take into account not just their competitors' reactions to entry in a particular location, but their reaction to every possible location choice the focal firm could make. Often, there is a very large number of location configurations and hence, significant computational challenges involved in modeling these decision. Studies have attempted to address such challenges through either a simplification of the problem (Mazzeo 2002b) or the addition of uncertainty about competitors' costs (Seim 2006). Other notable works in this area include the studies by Jia (2006) who extends this literature by considering contexts where firm decisions are not independent across locations and Orhun (2006) who extends Seim's (2006) framework by allowing for location-specific unobservables in the context of supermarket location choice. Zhu & Singh (2006) use a similar framework but relax the assumption that firms are symmetric and find that identities play a critical role in the discount retailing industry. Thomadsen (2007) explicitly examines the response functions of the various firms, in a large game context. He discretizes the strategy space (location) into a fixed number of choices, and then calculates the optimal response (embedding into this the second-stage price responses) to each discrete option in the strategy space. In a recent contribution to the literature on location choice, Sudhir, Datta & Talukdar (2007) explicitly incorporate agglomeration effects that arise from demand-side factors and argue that ignoring these effects biases the competitive strategic effects in entry models.

Product decisions. One of the earliest studies in the marketing literature on product characteristics choice was Horsky & Nelson (1992). In this paper, the authors study the optimal choice of product characteristics and prices for a new entrant in the automobile market. In extension of this work, Horsky, Misra & Nelson (2007) develop and illustrate a methodology to identify profitable new brand positioning and existing brand line repositioning and pricing for oligopolistic multi-brand firms. In this context a firm maximizes the joint profits over its brand line and each brand not only competes with the brands of other producers, but also cannibalizes (and is cannibalized by) other brands of the same producer. An added layer of complexity is the fact that the demand and supply curves commonly defined are not in the same multi-attribute space. Consumers evaluate the brands in a perceptual space while the firms produce them in a physical one. In their model the authors assume (and later estimate) that the consumer transforms appropriately weighted physical attributes into subjective perceptual ones, the factors that influence demand as well as costs. These estimated demand and cost functions then lead to a procedure to identify brand line profit maximizing physical space positioning and repositioning strategies which account for competitive price reactions. Recent additions to the literature are Mazzeo (2002b) who models the choice of quality along with firm entry decisions and Einav (2003) who investigates the timing of movie releases.

Draganska et al. (2007) take a first step toward exploring the product assortment strategies of oligopolistic firms in the ice cream industry by treating product choice as endogenous. Rather than assuming a reduced-form profit function to explain differences across markets in the observed product offerings as previous work has done, the authors specify a model of demand and product market competition and compute the implied equilibrium profits. The structural profit specification allows for counterfactual analyses to understand how changes, for example, in the nature of demand would affect the products offered in equilibrium.

There remain a number of open areas for research in this domain such as overall branding strategy (e.g., does a firm adopt a corporate brands or a house of brands), brand and line extension decisions and other such product related decisions, which so far have been assumed to be independent of competition.

Pricing. Traditionally pricing has been analyzed as a continuous control variable but there are numerous strategic pricing decisions that are of discrete nature. Ellickson & Misra (2007b) use a unique store level dataset to estimate the choice of pricing strategy (EDLP vs. HiLo) and the strategic interactions between supermarkets in this decision. The authors use a static game framework to investigate the influence that competitors' (expected) choices have on a focal store's choice of pricing strategy. Contrary to traditional wisdom, they find that stores tend to co-locate in strategy space even after accounting for observed and unobserved factors.

Mazzeo (2002a) and Singh & Zhu (2007) study the relationship between prices and market structure in the motel and auto rental industries, respectively. The approach in these papers essentially allows for a reduced-form selectivity correction for the market structure equilibrium outcome.

Another context where discrete choice models of firm decisions have been applied is that of tariff choice for products such as telecommunication services. Miravete (2002) explores the informational asymmetries that may drive tariff plan decisions for a monopoly local telephone service provider. Miravete & Roller (2004) estimate a static game where cellular service providers compete in tariff plans. Recent papers in this literature (e.g. Economides, Seim & Viard 2004, Narayanan, Chintagunta & Miravete 2007, Lambrecht, Seim & Skiera 2007) have focused on the demand side of these markets in their estimation, while exploring the impact of firm decisions through policy simulations using demand-side estimates. It would be logical to extend this work to study oligopolistic markets. This is challenging for a number of reasons. With some recent exceptions (e.g. Grubb 2007), the theory on competitive price discrimination relies on modeling approaches that are often difficult to adapt to particular empirical settings. On the empirical side, the challenges include a frequently very large set of tariff plans for whom prices need to be predicted in a tractable model and the limited variation in both the cross-sectional and time-series features of the tariff menus with which to identify the determinants of equilibrium prices. The theoretical work in this area also suggests the need for individual-level data with which to study variation in consumers' preferences for such services. With access to newer data on traditional telecommunication markets such as local and cellular telephone services and to data on other markets where consumers self-select the appropriate usage-based subscription plan, such as internet services, there is scope for considerable research on various questions in this area.

Promotions. There has been very little work that examines the interrelatedness of firms decisions when it comes to promotions. As mentioned earlier, Ellickson & Misra (2007b) examine the choice of pricing strategy of individual supermarkets. this pricing strategy, however, has direct implications for the amount and type of promotional activity the store engages in. In a recent paper Richards (2007) examines the decision of firms to offer promotions. While his framework is an approximation to the actual decision process (in that the decision variable is the number of items offered on promotion) it could be extended to allow the supermarket to chose *which* items to offer on promotion based on what promotions it expects competing stores to offer.

There is a vast literature on firm advertising decisions in the marketing and economics literatures. For example firms decide advertising spending strategies (e.g. choosing between pulsing vs. continuous spending), choose campaign strategies (e.g. to engage in comparative or competitive advertising), decide on media (TV, radio, print or combinations thereof) and so on. Similar examples of firm decisions can be found in other sub-areas such as sales promotion (e.g. do firms' offer coupons?) In most cases these decisions are also a function of what firms expect their competitors to do. This makes these decisions prime candidates for the application of the methods described in this paper.

5 Directions for Future Research

While the empirical literature on firm entry and location, product choice, and pricing decisions has received significant attention in recent years, there is scope for further research in a number of areas. There are important discrete marketing decisions where the level of analysis has not reached the above methodological level (in particular in terms of competitive oligopoly considerations) but may benefit from doing so in the future. Examples of such decisions include the decision whether a firm should employ its sales force or use outside agents, the choice of advertising media, and the selection of a particular branding strategy.

There is also a need for methodological advances in order to be able to model realistically the complexity of a firm's decision-making process. For example, a large part of the entry literature abstracts from firms' post-entry product market competition. To be able to do so, consumers are assumed to be homogeneous in their tastes for the product so that the demographic attributes of a potential market can proxy for demand. Attempts at incorporating a more realistic demand model and accounting for firms' pricing decisions have had to make strong assumptions about the structure of demand to keep the model tractable. The current state of the literature thus rules out studying more realistic and interesting demand settings. For example, could a demand model incorporate consumer's bounded rationality, as has been shown to be relevant to consumers' choice of subscription services, to investigate how firms may exploit suboptimal consumer behavior in their entry and product design choices?

A further area that needs attention is the development of more efficient estimation algorithms. While approaches exist for the (relatively) fast estimation of both static and dynamic games, they are not as useful for counterfactual analyses. This limitation hampers our ability to understand the implications of policy changes for the equilibrium outcomes.

In sum, this paper outlines the methods and applications related to the nascent area of empirical discrete games in marketing. We provide a broad discussion of the problems and opportunities that exist in this research domain and hope that this review will spur new developments in the years to come.

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