# *Thinking Strategically: Game Theory*

*This note introduces* ***game theory,*** *a tool for framing and analyzing strategic situations. Prior to class, please prepare your answers to the problem on pages 8-9—cognizant of the midterm, the problem is mercifully short. In class we will play some games to illustrate key ideas, so please bring a few dollars of your own.*

*Game theory* is a tool for making competitive sense of some of the most common strategic situations in the business world. Examples include price wars, new product introductions, strikes, negotiations, and divisional relationships. Because payoffs in these settings are inter-dependent, a manager's optimal decision depends on what she expects others to do.

Good managers attempt to influence the behavior of others by systematically evaluating the variables subject to their control, and using these variables to influence others expectations. To do so, they need a framework that explicitly considers (and anticipates) the actions of others. Game theory provides that framework.

## *What are management strategy "games"?*

There are two types of management games. First, a *cooperative game* is one in which players can write binding (i.e., legally-enforceable) contracts with each other. We use "cooperative game theory" or "bargaining theory" to understand these games.

Second, there are "non-cooperative" games. A *non-cooperative game* is one where no binding (i.e., legally-enforceable) contracts can be written by the players in the game. The basic problem with playing the cooperative strategy in such a setting is that you put yourself at the "mercy" of others, and they have an incentive to exploit this position.

This and the next session will concentrate on teaching you the basics of non-cooperative game theory.

When examining a strategic situation as a game, we consider three key elements:

1. The list of individuals or parties involved, called the *players;*
2. The *rules of the game*: Who has what options when, and with what information;
3. The *payoffs* that indicate how well each player fares for every possible outcome.

Game-theoretic reasoning is a way to predict the strategies players will select. For example, in Session 3 we studied strategic capacity decisions in an oligopoly. The logic underlying the Cournot solution is that you want to choose your output level based on the knowledge a rival is thinking about its decision the same way. The market *equilibrium*—that is, the stable outcome—is obtained by finding the intersection of the two reaction functions. This same logic can be usefully applied in many other, quite different strategic situations, as we shall see.

## *Games and their Representations*

There are two basic types of "models" we use to represent games and analyze strategic situations. One is called the *strategic form*, and the other is called an *extensive form*.

Figure 1 is an example of a strategic form game. This consists of a table that indicates:

- Who the players are;
- What actions each player can choose from;
- The payoffs to each player for each outcome.

Strategic form games are useful when there are two players, in which case we list one at the top of the table (*Allied)* and one on the left side (*Barkley*). We then create a *column* for each possible action that Allied might choose, and a *row* for each possible action that Barkley might choose. In Figure 1, Allied can choose between three possible prices of \$0.95, \$1.30, or \$1.55, which are listed as the column headings. Barkley's possible actions are \$1.00, \$1.35, or \$1.65, and are listed as row "headings" along the left side of the table.

Payoffs in strategic form games have a particular convention. This is to avoid any confusion when analyzing (and communicating about) the game. The convention is:

*Payoffs are listed as a pair: X,Y where X is the payoff to the row player and Y is the payoff to the column player.*

So, for example, if Barkley chooses \$1.35 and Allied chooses \$1.30, the payoff to Barkley is 8 and the payoff to Allied is 2. See Figure 1.

| | | *Allied's price* | | |
|---|---|---|---|---|
| | | *$0.95* | *$1.30* | *$1.55* |
| *Barkley's price* | *$1.00* | 3, 6 | 7, 1 | 10, 4 |
| | *$1.35* | 5, 1 | 8, 2 | 14, 7 |
| | *$1.65* | 6, 0 | 6, 2 | 8, 5 |

FIGURE 1. *A Strategic Form Game*

The alternative way to represent a game is the *extensive form*. This consists of a diagram with nodes and arrows, and labels to indicate who is making what decision at each stage of the game. Extensive form games are quite similar to the decision trees of MGEC 603, except that they represent decisions of two (or more) players.

The extensive form (or *game tree*) is particularly useful for representing situations when the players move sequentially, rather than simultaneously. For example, suppose that the payoffs to the pricing game between Allied and Barkley are the same, but that Barkley is able to move first in choosing its price *and Allied will know Barkley's decision when Allied chooses its price.* This situation is usefully represented in an extensive form, as shown in Figure 2. Again, the payoffs corresponding to each outcome of the game are a pair, where the first element is the payoff to the top (first) player.

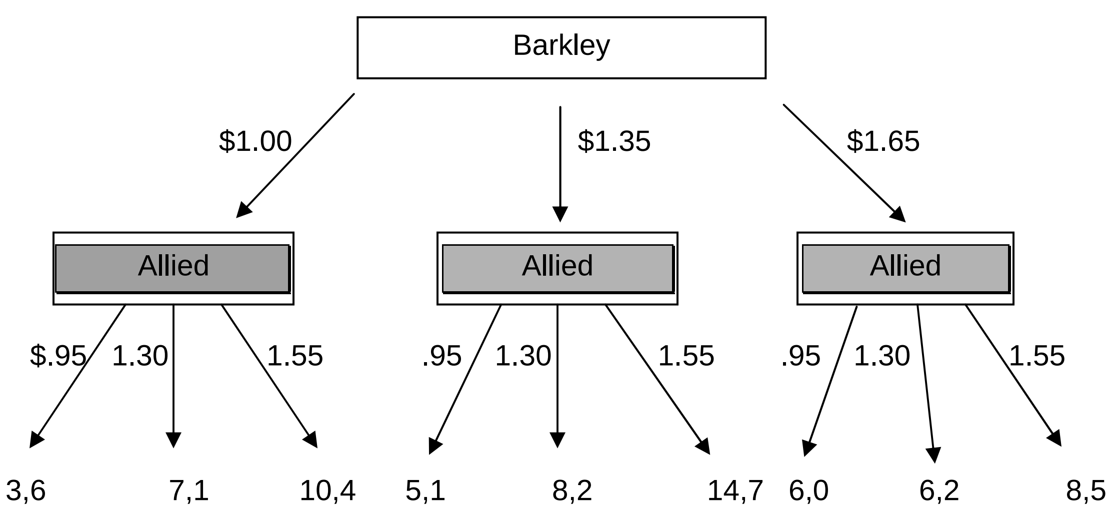



FIGURE 2. *An extensive form game for sequential moves*

## *Analysis: Two Solution Methods*

There are two ways of "solving" games that we will employ. By "solving", we mean coming up with a recommendation of how one (or both) players should act in the situation being modeled by the game. This requires careful attention to the payoffs, as well as who knows what and when they know it, and so on… in short, thinking carefully about the structure of the situation. The two solution methods we will consider are *dominance analysis* and *Nash solutions*.

### Dominance Analysis

In some situations, a player will have an option that is either (1) clearly the best thing to do *regardless* of the other players' actions; or (2) is clearly *not* a good thing to do regardless of the other players' actions. When the first case holds, that best strategy is (obviously) a good recommendation for what that player should do. We then say the player has a *dominant strategy:* There is a choice for this player that dominates (is better than) all other choices she might make, regardless of the other players' decisions.

Similarly, when the second case holds, we have a recommendation for what the player should *not* do. This is called a *dominated strategy:* There is a choice for this player that is worse than its other choices, regardless of what the other players do. When one or more players have a dominated strategy, we can often iterate back-and-forth by ruling out the actions that are clearly dominated. This will sometimes leave us with a single best strategy, and a strong recommendation for what the player should do.

The game in Figure 1 is an example of a strategic situation in which there are dominated strategies. Follow the logic using Figure 1: If we look at Barkley's payoffs from choosing a price of $1.00, we can see that they are worse than if Barkley chose $1.35 *regardless of what Allied decides to do.* So we eliminate $1.00 from consideration in thinking about what Barkley should do.

Now, time for the interactive part: Suppose that Allied follows the same logic we just did, and concludes that Barkley will never price at $1.00. Then Allied should make its decision considering only the bottom two rows of Figure 1. Now look closely: For Allied, a price of $1.55 beats the payoff associated with either a price of $.95 or $1.30, given that Barkley would not want to set its price at $1.00. This logic leads us to the recommendation that Allied set its price at $1.55.

Now back to Barkley once again. If it can carry through this logic this far as well, then Barkley is left with a choice between $1.35 and $1.65 *knowing* that Allied will set its price at $1.55. What is the best thing for Barkley to do? Looking at the two lower cells in the far right column, we see that Barkley would maximize its profit choosing a price of

$1.35, for a payoff of 14. Our conclusion? If Allied and Barkely both know enough not to choose a dominated strategy, and *expect that the other is smart enough not to choose a dominated strategy either*, then Barkley will price at $1.35, Allied at $1.55, and they will make 14 and 7, respectively.

***Remark***. In many games, this sort of back-and-forth elimination of dominated strategies will not lead to a *unique* prediction about what each player should do. Rather, what dominance analysis usually provides is a way of "narrowing down the game," by pruning out only some (but not all) of the players' choices from consideration. This will typically make a problem more manageable to solve, but still leave us with the need for a different solution method to complete the analysis. That brings us to our second solution method: *Nash solutions.*

## Nash Solutions

The most well-known and widely-used concept in game theory is called a *Nash equilibrium*. It is named for the famous mathematician John Nash, who proposed it as a solution to competitive situations that are amenable to game-theoretic analysis. A Nash solution is a set of strategies such that no player has an incentive to change its strategy unilaterally, given the strategies chosen by the other players. It is a solution of mutual best responses: my choice is optimal *given* others' choices, and vice versa. Nash's idea is precisely the same as the mutual *no regrets* logic employed in our analysis of Cournot quantity competition in Session 3.

The concept of a Nash—or mutual no regrets—solution is best conveyed by an example. Figure 3 shows a pricing game between two firms.

| | | *Firm 1* | |
|---|---|---|---|
| | | Price Low | Price High |
| *Firm 2* | Price Low | 1, 1 | 10, 0 |
| | Price High | 0, 10 | 5, 5 |

FIGURE 3. *A Pricing Game (aka, the Prisioners' Dilemma*)

Now, if we were to rank-order the four possible payoffs from *one* firm's perspective, we would select the following order:

1. For each firm it is better to price low while your rival is pricing high.
2. Both price high
3. Both price low
4. You price high but your rival prices low.

*What is the Nash equilibrium here?* Stare at the payoffs in Figure 3 a bit. If Firm 2 prices high, then Firm 1 would prefer to price low. Similarly, if Firm 1 prices high, then Firm 2 would prefer to price low. So any (high, low) outcome is not mutual no regrets: One of the two firms would prefer not to have chosen a high price. The only outcome in which both firms have no regrets is (low, low): If Firm 1 chooses low, the other would not want to change its decision to high after learning what happened, and vice versa. The strategy of choosing the low price by each firm is the Nash equilibrium of this game. A Nash solution is always a stable one, in that neither player would prefer to unilaterally change its action.

***About this game.*** The payoff structure in Figure 3 is a famous problem, known generally as *the prisoners' dilemma.* It is commonly used to help understand why, even if cooperation can make a profit "pie" bigger for all of the participants, the players often do not cooperate. In Figure 3, the cooperative outcome is for both players to price high, maximize the value of the market between them, and split the gains equally at five apiece. The basic problem with playing this cooperative strategy from an individual player's standpoint, however, is that you put yourself at the "mercy" of the other firm—and the other firm has a strong incentive to exploit this position and choose a low price instead. This is precisely the same logic we used to explain why collusion is usually not stable in Session 4.

In business, this type of strategic situation is fairly common. For example, consider two firms selling a similar product. Each can advertise, cut price, or improve quality, which may improve its own profit and decrease the profit of its rival (holding fixed the actions of the rival). But increased advertising by all firms, or price-cutting by all players (like Boeing and Airbus), can sometimes decrease everyone's profits. Or consider countries that impose tariffs between them, making them all worse off in the end.

In sum, the logic in this simple game helps explain behavior exhibited by managers in many market settings. Again, the models explicitly account for the behavior of other players.

## *Example*

The main issue facing a manager in committing firm resources to a cooperative strategy (like deciding to produce only 6 in the game in Figure 4) is that if the firm cooperates and other player(s) do not, then the manager pays a large penalty in terms of payoffs. This is the logic behind why collusion is unstable in market games, at least in one-shot settings.

Consider the capacity choice game between two rival firms shown below in Figure 4. What is the Nash equilibrium of this game?

|  |  | *Firm 2's capacity* |  |  |
|---|---|---|---|---|
|  |  | *6* | *8* | *9* |
| *Firm 1's capacity* | *6* | 12, 12 | 0, 20 | –6, 21 |
|  | *8* | 20, 0 | 4, 4 | –4, 3 |
|  | *9* | 21, –6 | 3, –4 | –6, –6 |

FIGURE 4. *A duopoly capacity choice game.*

This game should look familiar. This is a strategic-form representation of the duopoly game we played in class session 3, pared down to the three quantity choices that students usually focus on. I used the demand and cost information from session 3 to calculate the profits of each firm listed in the table, which depend on each firm's quantity choice. (You could, of course, expand this out to at 25-by-25 table to consider all possible quantities, but this 3-by-3 version is sufficient to make the point about what outcome is stable here). If firm 1 is planning to choose a capacity of 6, *and the other player anticipates this,* what is the most profitable thing for firm 2 to do?

Would playing this game again and again with the same rival improve matters? Could the two firms achieve a mutually more profitable outcome in that way?

## *More Examples*

Game theory is used to analyze the essential features of strategic competition in a wide variety of business settings, including product positioning decisions between firms in the same market, pre-emptive investment to deter market entry, and incurring (short-term) losses to build a reputation for toughness and deter future competitors. Here are two (simplified) examples.

**A Product Positioning Game.** What is the Nash solution in the game below?

|  |  | *Krullogs* |  |
|---|---|---|---|
|  |  | Crispy | Sweet |
| *Junk Food General* | Crispy | –3, 1 | 2, 2 |
|  | Sweet | 5, 5 | 1, –3 |

This product positioning problem is an example of a *coordination game.* In coordination games, there may be more than one Nash equilibrium. The issue is which one to play, if both players must make their choice simultaneously. In this coordination game it can be hard to ensure a "good outcome", even though there is no real conflict of interest between the two players. Both would like to coordinate on an outcome in which they avoid head-on competition and do not select the same product attribute.

**An Entry Game.** As managers you will face many coordination games. In many settings, firms would like to coordinate to avoid disastrous outcomes but they have strong—and opposing—preferences over what to coordinate on. What is (are) the Nash solution(s) here?

| | | *US Scare* | |
|---|---|---|---|
| | | Enter | Don't Enter |
| *Untied* | Enter | –5, –5 | 15, 0 |
| | Don't Enter | 0, 15 | 0, 0 |

This type of payoff structure is typical of inter-dependent market entry decisions between firms, if total demand is not large enough to support both players and neither has already moved first. As you may suspect from thinking about how you would play this game, a firm's reputation has a great deal of impact on the outcome in situations like this. What is the Nash equilibrium in this game, if you only played it once?

Students might recognize the payoffs in this game by another name: "chicken." Chicken is a pastime of American kids who ride bikes straight at each other, trying to see who will veer off first and get the undesirable reputation of a chicken. Both get this reputation if both veer off. If neither one does, they crash and both get a trip to the hospital. It's effectively the same game in business when you enter a new market (except you play it with other peoples' money, of course).

# *Problem to Prepare for Class*

*The examples in this note were chosen to be simple, in order to expose concepts clearly. Many directly managerially-relevant applications will come in the next session. Still, at this point the following problem should highlight how game theory helps us understand things in new ways.*

*Prior to class, please prepare your answers to the problem below. As usual, you may work with others if you wish.*

## The E.U. Airbus Subsidies

When Airbus Industries got its start, it was believed that there would not be enough total demand for two major aircraft manufacturers (viz., Airbus and Boeing) to both turn a profit developing a new mid-size aircraft line (roughly 4000mi range with 150-200 passengers). Rather, because aircraft development involves such high fixed costs, world demand would make it economical for only one firm to produce a new aircraft in this size range. The inter-dependence of the two firms' profit and decisions in this product line can be represented by the payoffs of the following game (payoffs in billion $):

| | | *Airbus* | |
|---|---|---|---|
| | | Produce | Don't Produce |
| *Boeing* | Produce | –10, –10 | 100, 0 |
| | Don't produce | 0, 100 | 0, 0 |

(a) If Boeing has a head start in the development process for this line of aircraft, then what would be the outcome of this game? If Boeing does not have a head start, then what (if any) outcome *of this game* would you expect?

As is usually the case in high-capitalization industries, there are other players involved. Airbus is partially owned by four nations (France, Germany, Britain, and Spain), and they would prefer that Airbus produce the new aircraft. Suppose that these governments, through the E.U., *commit to a subsidy of $20 billion* before Boeing has committed itself to produce. Regardless of what Boeing decides to do, this subsidy would raise Airbus' payoff by 20 (billion $) if Airbus decides to produce the new aircraft (however, Airbus would not receive the subsidy if it decides not to produce the new aircraft).

(b) Draw a new strategic-form game incorporating the subsidy into Airbus' payoffs. How does this change the outcome of the game? Why?

(c) From the standpoint of the European governments' that provided the development capital (i.e., the subsidy), why does subsidizing Airbus yield such a high return on their investment?

# *Application: Wireless License Auctions*

One important application of game theory to business practice is the design of auctions. Consider the attached article about the US government's first wireless (mobile phone) license auctions, held in 1995. Analyzing bidding strategies in these auctions using game theory was important both to the firms involved, and to the experts hired to design the auctions rules in the first place. We'll see more of this when we discuss auctions in detail in session 8.

## Game theory in action: designing the US airwaves auction

In 1993 the US Congress decided to auction off licences to use the electromagnetic spectrum for personal communication services. This involved selling off thousands of licences with different geographic coverage and at different spectrum locations.

Auctioning off the licences was a break with the tradition of direct licence allocation to those with a bigger "need." It required the Federal Communications Commission to set up a mechanism capable of efficiently allocating licences to the bidder most valuing it. Game theory (and game theorists) played an important role in both the design of the actual auction mechanism used and in advising bidders on optimal bidding strategies.

Auctions are, a priori, an ideal method for allocating goods to those who place a higher value on them, as these people are likely to make the highest bid. However, research by game theorists has shown that the design of the auction matters, both for the efficiency of the allocation — does the good go to the person who values it most? — and the revenues earned for the seller. In this particular instance, congress had asked the FCC to ensure that the spectrum was used in an efficient and intensive way, rather than simply to maximise auction income for the Federal Government.

According to an account in the Journal of Economics Perspectives, by two of economists involved in the design (Preston McAfee of Texas A&M and John McMillan of University of California, San Diego), the designers considered the existence of complementarities between the licences the most important threat to efficiency in this particular context.

For the bidder, the value of each individual licence depends to a large extent on whether another licence has been obtained so that several licences can be grouped together to form a coherent region. The auction design needed to allow for the coherent aggregation of licenses, so that a bidder would not find himself bidding for a licence as a part of a whole to discover that he is in fact awarded an incoherent entity of much smaller value to him or her.

Following the advice of several economic theorists employed by the bidders, including Stanford economists Paul Milgrom and Robert Wilson, plus McAfee and McMillan, the FCC opted for a novel design: a simultaneous ascending auction, in which the bidding for all the licences remained open as long as bidding in any of the licences remained active. The aggregation of licences was facilitated by the fact that bidding and the observation of the bids, were simultaneous.

For all its advantages, the simultaneous ascending auctions involved an important risk of implicit collusion between rival bidders. To avoid this problem, the identity of the bidder would remain hidden until the auction concluded. But it was still possible, for a bidder to find ways to signal their intention in order to ensure allocation of the preferred licence at a low cost. In fact, as subsequent portions of the spectrum were auctioned off, complaints about this type of behaviour increased. For example, Mercury PCS, a US telecoms operator, was accused of highlighting its interest in winning a specific licence by ending the bid amount in January 1997 with the postal codes of the particular city in which it was interested.

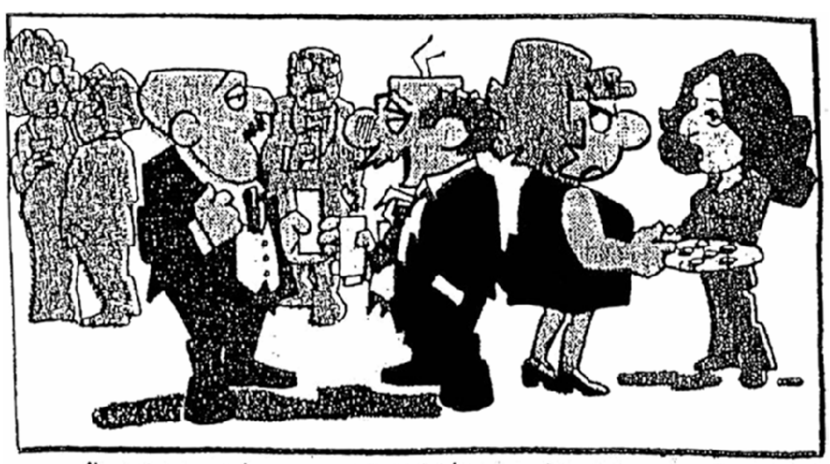
On the bidders side, the consultants to the bidders have not made public their recommendations. We can only speculate that game theory was used by bidders both to enhance the rationality of their bidding and to understand and influence the bidding behaviour of the rival bidders.

First, game theory could introduce a higher degree of rationality in the bidding process by helping to design optimal bidding strategies. For example, game theorists have long understood that in auction settings bidding as much as one would be actually willing to pay for a good could lead to what is known as the winner's curse: imagine that a licence will be in fact equally valuable to any firm, but that each firm has a different opinion about how valuable it is likely to be. Clearly, the winning bid is most likely to be from the most optimistic firm. But the most optimistic firm's estimate of the value of the audio waves will be biased upwards — it will be too high— even when the estimate by each individual bidder is unbiased. In fact, if the highest bidder wins without taking into account this effect, he will overpay, and winning will be a curse. Developing an algorithm for bidding that takes into account this curse requires the use of game theory.

Game theory could also be used by a bidder to understand the incentives of rival bidders and formulate strategies capable of altering their behaviour. In particular, if one could credibly commit to winning a licence, one could win the licence at zero cost, as the incentives for the rivals to bid when they know they are not going to win the licence, are likely to be low. An actual example of this could be the allocation in the April 1997 sale of wireless date frequencies, of the licences for several cities like Minneapolis, for $1.

To sum up, by identifying the individual incentives of each player in each auction design, game theory helped the designers and consultants in understanding the impact of the rules of the game on the behaviour of the actors. As McAfee and McMillan put it in their account: "The role of theory is to show how people behave in various circumstances, and to identify the trade-offs involved in altering those circumstances."

## *Amusements: The Lockhorns on Nash Equilibrium*



"LORETTA'S DRIVING BECAUSE I'M DRINKING, AND I'M DRINKING BECAUSE SHE'S DRIVING."