Risk Aversion and Risk Sharing

*This note introduces **expected utility, risk aversion**, and how **markets transfer (share) risk**. Prior to class, please prepare your answers to the problem that begins on page 9.*

We made decisions daily. Most require little effort and the choice is simple (e.g., what should you eat for lunch?) Often, however, we must make decisions that demand more careful thought because the decision involves considerable uncertainty (e.g., how much to save for retirement, or whether to invest $40 million in a new firm). In such situations, we want to think harder about choices and their consequences.

This note presents **one** formal approach for cutting through complicated—meaning uncertain and/or imperfectly understood—economic decisions. The approach is called ***expected utility maximization***. This theory also helps us understand how and why markets efficiently transfer risk away from risk-averse investors, homeowners, entrepreneurs, creditors, and others. Since many markets exist for this purpose—most financial securities, insurance, options and futures, for example—it is worth our while to know the economic principles behind how these markets work.

Theory

Expected monetary value (EMV) is one preference scheme you encounter in several core classes. It is obtained by multiplying each possible outcome by its probability, and summing these products over all possible outcomes. For instance, consider a gamble that pays either $50 or $0, with probabilities .6 and .4, respectively. The expected monetary value of the gamble is

$$\text{EMV} = \$50 \times 0.6 + \$0 \times 0.4 = \$30 .$$

The EMV is also called the mean of a gamble. It is *sometimes* a useful guide for thinking about how much an individual would be willing to pay to take on (or, in other contexts, to avoid) a risky gamble. Typically, however, EMV is not so good at predicting what people will pay to acquire (or to avoid) a larger risky opportunity. For instance, suppose I ask which of two options you prefer to have:

Option A: A gamble that pays $50,000 or $0, with chances .6 and .4 respectively;

or

Option B: A payment of $30,000 for certain?

When asked, most people have a strong preference for Option B. And, if the scale is increased into the millions of dollars (by multiplying everything by another $1,000), then most peoples' preference for the sure thing becomes quite strong. When a person strictly prefers to have the EMV of a risky gamble (for certain) to facing the random outcome of the gamble, we say the individual is *risk averse.* An individual who is indifferent between taking a gamble and its having its EMV for sure is *risk neutral.* And, in some cases, an individual might prefer a gamble to its EMV; we call this *risk seeking.*

The fundamental problem with using EMV as a predictor of how much people would pay to take on (or to avoid) a risky situation is that *EMV does not account for a person's aversion to risk.* Thus, using EMV is often a poor predictor of the decisions people will make in uncertain economic situations—such as acquiring insurance, or buying a risky asset. To do a better job at predicting what people will do, we need a better theory.

Expected Utility

The standard economic model of how people make economic decisions facing risk is called **expected utility maximization**. It allows the decision maker to reflect the impact of risk. Usually people are averse to risk, but preferences do vary. Expected utility still considers a probability-weighted average of what might happen in a risky situation, but instead of using monetary values for outcomes, it uses *personalized* outcomes called *utility values*. The idea is that the shape of this utility function will account for the decision maker's aversion to risk.

The first part to consider is the utility function. The assumption here is that money gives us a certain amount of satisfaction, or "utility". The **utility function** is a device for showing how much satisfaction we get from different possible levels of wealth. An example of such a utility function is shown in Figure 1. The horizontal axis shows possible levels of wealth, and the vertical axis shows the corresponding level of utility. There are two key features of this utility function to note.

- The slope of the function is up. This means more wealth gives more utility. (No surprises there—if you disagree, you can simply give me some money and we will both be happier).

- Second, the curve slopes upward at a decreasing rate (it bends over as it goes up), which means it is *concave.* This implies that the incremental (marginal) satisfaction derived from each extra dollar declines the wealthier we become. Again, that sounds reasonable, and this concept is known technically as "diminishing marginal utility."

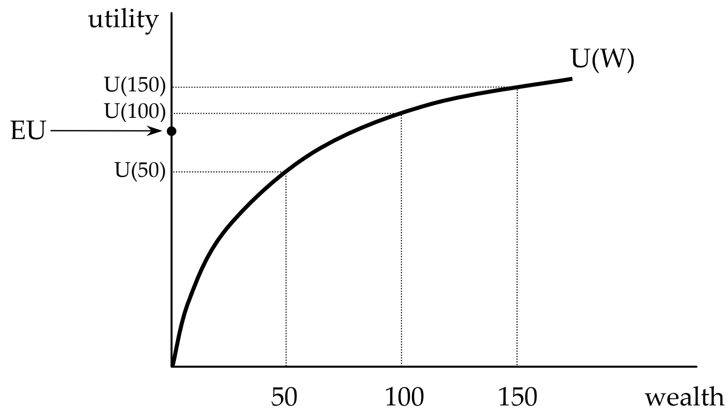

FIGURE 1. *A Utility Function for a Risk-Averse Person*

Maximizing Expected Utility

Now the expected utility model states that when faced with uncertain outcomes, we do **not** take the EMV of our wealth. Instead, we consider the expected value of our *utility of wealth*. The formula is:

$$EU = \Sigma \; p_i \times U(W_i)$$

where W_i denotes possible levels of wealth and p_i is the probability of obtaining that wealth. We will illustrate how expected utility works via the use of simple diagrams and situations first. Then, we will adopt a more mathematical approach which lends itself to solving more realistic problems.

Consider the following situation. You initial wealth is \$100. You are offered a lottery where you can win \$50 with a probability of .5, or lose \$50 with a probability of .5. Figure 1 shows the EU of playing the lottery is lower than the utility of not playing. That is, the EU you receive from playing the lottery is lower than the utility you receive from keeping your \$100, and not playing. To estimate the lottery's EU, you calculate EU by taking the weighted average of the utilities of the two possible outcomes:

$$EU = 0.5 \times U(150) + 0.5 \times U(50)$$

In Figure 1, this is the half way point between U(150) and U(50) (since the probability of each outcome is .5) on the vertical axis as shown. Thus EU from gambling is less than U(100), which is the utility of not gambling and keeping your $100. Concave utility implies you would prefer the EMV for sure ($100) over taking this gamble.

At an intuitive level, a risk-averse person is willing to sacrifice some income to avoid risk. That is, he or she is willing to pay someone to assume the risk that he or she doesn't want. For example, people are willing to pay insurance premiums that include a markup over the expected loss (to cover insurer profit and expenses and agents' commissions) to off-load a risk to an insurer. The expected monetary value of the buyer's wealth from paying these premiums is negative, meaning that a person's expected wealth actually *decreases* when they buy insurance. But this decreased expected wealth is worthwhile to a risk-averse buyer, who will now be compensated if an adverse event occurs.

A second example lies in bond returns (and other assets returns). Corporate bonds have higher expected yields than U.S. government bonds. People still buy U.S. government bonds knowing they are accepting a lower yield; buyers are willing to accept the lower EMV of U.S. government bonds because of their low risk. Put the other way around, people need higher yields on corporate bonds to compensate them for the higher risk. Both the wide use of insurance and government bonds are evidence that many individuals are risk averse.

The Risk Premium

Some important terminology goes along with the phenomenon of risk aversion. Given any gamble, we can imagine asking an individual who faces it: Would you prefer the gamble, or $X for sure? Depending on the size of X and the person's wealth, the individual might prefer the gamble or the sure thing. When X is set at the precise amount that makes the individual totally indifferent between the gamble and the sure thing, we say that $X is that individual's *certain monetary equivalent* for the gamble, or *certainty equivalent* (CE) for short.

In this terminology, risk aversion means a gamble's certainty equivalent is less than its expected monetary value, or CE < EMV. Risk neutrality is CE = EMV; and risk-seeking behavior is CE > EMV.

Assuming risk aversion, so that CE < EMV, the difference between the certainty equivalent and the expected monetary value, or EMV – CE, is called the individual's *risk premium* (RP) for the gamble. The larger the risk premium, the greater is the distance between the gamble's EMV and the subjective value of the gamble to the individual, CE. Intuitively, a higher risk premium means the individual is willing to forgo a greater expected gain in order to avoid the risk.

Evaluating Expected Utility

To solve problems, we need to know what mathematical formula the utility function should take. Risk aversion is represented by various types of formulae. For example:

$U(W) = W^a$ where *a* is some number between 0 and 1,

or

$U(W) = \log(W)$.

When facing a risky situation, maximizing expected utility amounts to doing the algebra that goes with pictures like Figure 1. Here are a few examples.

Example 1

You have a utility function over wealth given by $U = W^{.5}$, and an initial wealth level of \$100. This wealth level will be reduced to \$40 if your house burns down (i.e., the house is worth \$60 and you will lose this if you have a fire). The probability of the fire is 25%. However, you can insure against the possibility of a loss by paying \$15 now. Thus, if you insure, you will have certain wealth of \$85. Note that in this case your wealth if you buy the insurance (\$85 for certain) is the same as the expected monetary value if you don't (EMV = \$85). Should you insure? What is your risk premium?

If you acquire insurance, you have certain wealth of \$85 so your expected utility is just $U = 1 \cdot (\$85)^{.5} = 9.22$ utils. Alternatively, if you do not acquire insurance your expected utility becomes

$$EU = .75 \cdot (\$100)^{.5} + .25 \cdot (\$40)^{.5} = 9.08 \text{ utils.}$$

Your expected utility is higher if you buy the insurance. Figure 2 shows why the rational decision is to insure, and that the utility function $U = W^{.5}$ is concave. The EU with no insurance is $.75 \cdot U(100) + .25 \cdot U(40)$, and is pictured as ¾ of the distance up from U(40) toward U(100) on the vertical axis. This is less than the expected utility if you buy insurance, which is U(85).

Your certainty equivalent for this risk is the amount of wealth that gives you the same utility as you expect to have facing the gamble, 9.08. Solving $U = W^{.5}$ for W when U = 9.08 gives W = \$82.47. This is your certainty equivalent. Since the EMV of your wealth is \$85, your risk premium is RP = \$85 – \$82.47 = \$2.53. In words, you would be willing to pay the difference between your initial wealth (\$100) and the EMV (\$85), *plus* a risk premium of \$2.53, in order to avoid the financial risk of your house burning down. But not one penny more.

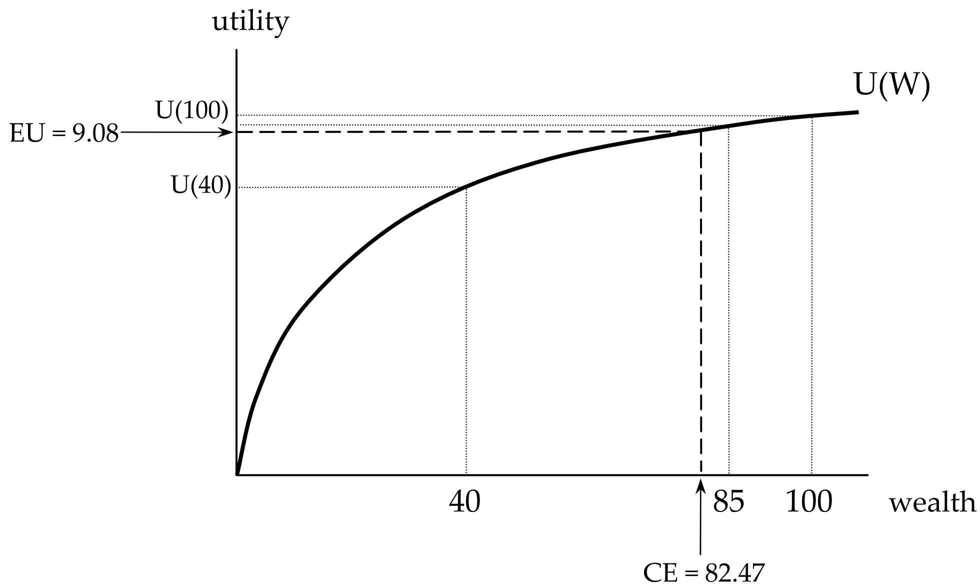

FIGURE 2. *Expected Utility and the Certainty Equivalent in Example 1*

A WARNING: Do not make the following mistake when calculating EU for risky situations: <u>Do not take the expected value of wealth</u> (which is .75(100) + .25(40) = 85) and then the square root of 85 which is 9.22. This is not the expected utility, it is the utility of the expected outcome. And the two are definitely not the same. In fact, this way of calculating EU actually ignores the effect of risk altogether.

Take the expected value of the utilities. DO NOT take the utility of the expected monetary value.

Example 2

You have initial wealth of 1000, but face the prospect of total loss of your wealth from an insurable loss. The probability of this loss is 0.2. Assume again your utility function over wealth is $U = W^{.5}$. What is the maximum you are willing to pay for insurance, and your risk premium?

Without insurance, your expected utility is $EU = 0.8\times(1000)^{.5} + 0.2\times(0)^{.5} = 25.2982$. With insurance at a price of P, you have $EU = (1000 - P)^{0.5}$. Now, choose a level of P that leaves you indifferent between insuring and not. This means finding the value of P that equates your expected utility with and without insurance:

$$25.2982 = (1000 - P)^{0.5}$$

Solving gives P = 360. Since any price greater than 360 will make the no-insurance option yield a higher expected utility, the maximum you are willing to pay is 360.

Your certainty equivalent for this risk is the wealth that gives you utility 25.298, or CE = $(25.298)^2$ = 640. Thus, your certainty equivalent is just your initial wealth *minus* the maximum you would be willing to pay to avoid this risk. That should make sense to you: the certainty equivalent, after all, is the wealth level that leaves you indifferent to having it for sure and facing the risk uninsured. So, facing a risk of a loss, you would pay an amount that leaves you with your certainty equivalent (or more) to avoid it.

Last, since the EMV of your wealth if you have no insurance is EMV = \$0(.2) + \$1,000(.8) = 800, your risk premium is RP = 800 – 640 = 160. That's a big premium over the EMV to pay just to avoid this risk. But, on the other hand, we're considering a 20% chance of losing your entire wealth in this example, and a risk-averse person might pay a lot to avoid that possibility.

Example 3

We often have choices, as in health insurance or car insurance, about *how much* insurance protection to obtain. For example, we can "fully insure" (all losses are covered in full), or we can insure with a deductible—we pay the first \$X of any loss and the insurer covers anything above \$X. Finally, in some markets we can insure for a percentage of loss, called "co-insurance". Both deductibles and coinsurance are examples of *risk sharing.*

Suppose that your utility function over wealth is $U = 2W^{0.5}$, and your initial wealth is W = \$130. You face a risk that involves a loss of \$20 with probability .3, and a loss of \$100 with probability of .2. You will have no loss at all with probability .5.

An insurance firm offers you the following three possible policies:

1. A *full insurance policy,* at a premium of \$32.5;
2. A *partial insurance policy* with a deductible of \$20, at premium of \$20;
3. A *partial insurance policy* with a coinsurance rate of 61.54%, at premium of \$20.

Which, if any, of these policies should you buy? Note that we can also choose not to insure at all.

The following table lays out the insurance payments under each policy for each level of loss. It also shows the wealth remaining to the policyholder after suffering the loss and receiving any insurance compensation.

Loss Amount	Prob.	Wealth with no insurance	Policy 1: You are paid	Wealth with Policy 1	Policy 2: You are paid	Wealth with Policy 2	Policy 3: You are paid	Wealth with Policy 3
$0	.5	$130	$0	$97.50	0	$110	0	$110.00
$20	.3	$110	$20	$97.50	0	$90	$12.31	$102.31
$100	.2	$30	$100	$97.50	$80	$90	$61.54	$71.54
EMV=26		**EMV=104**		**EMV=97.5**		**EMV=100**		**EMV=100**

Using the results in the table, we can work out the expected utility under each insurance policy choice, as well as the expected utility if you buy no insurance at all:

1. Full insurance policy: $EU = 2\times\{.5\times(97.5^{0.5}) + .3\times(97.5^{0.5}) + .2\times(97.5^{0.5})\} = 19.75$
2. Deductible policy : $EU = 2\times\{.5\times(110^{0.5}) + .3\times(90^{0.5}) + .2\times(90^{0.5})\} = 19.97$
3. Coinsurance policy: $EU = 2\times\{.5\times(110^{0.5}) + .3\times(102.31^{0.5}) + .2\times(71.54^{0.5})\} = 19.94$

And with no insurance, $EU = 2\times\{.5\times(130^{0.5}) + .3\times(110^{0.5}) + .2\times(30^{0.5})\} = 19.88$. Which choice maximizes your utility? It is a near thing, but the deductible policy is the best choice here.

Question: In this example, why is your utility lower with full insurance than with no insurance at all?

Back to Markets and Economic Analysis

Consider the numbers in Example 3 once more. If you ran an insurance company, could you offer this consumer a full insurance policy and make a profit if your "loading cost" is $2.00 per policy? (Loading cost is the labor, materials, and capital costs of managing your insurance business.) You have to offer greater utility to this risk-averse buyer than that now available on the market with the deductible policy. What full insurance premium would you charge? What is your expected profit on the policy? If your firm can be copied by another entrant, what happens to premiums and then to your profits?

Pooling Risks

One way risk-averse individuals can benefit is by joining together and sharing risk. Essentially this is what insurance companies do. If the risk between group members is independent, (i.e., the likelihood of an event affecting me is independent of it affecting you), then individuals lower their insurance costs by pooling together. This reduction in costs is due to the law of large numbers—if the probability of an auto accident is .001, and you are insuring a pool of 1 million drivers, then those drivers will be involved in approximately 1,000 accidents per year (with very little variation, too—the actual number of accidents will typically be about 1,000, plus or minus 30 or so). So while the probability of an accident for individuals varies greatly, the total number of accidents in a large pool of drivers is quite stable. Other common pooling arrangements used for reducing risk of members are joint ventures and partnerships.

Putting This in Perspective

The expected utility maximization model is robust enough to capture the phenomenon of risk aversion. Moreover, it does not *require* risk aversion: Depending on the shape of the individual's utility function, behavior that is risk neutral or risk seeking can be accommodated. (A risk neutral person has a utility function that is linear—a straight upward-sloping line—while a person who is risk seeking has a utility function that bends *upward* as it rises instead of bending over as it rises). Risk aversion is, of course, a matter of individual taste, opportunity, and experience. Everything else held equal, a student on a tight budget is more likely to be risk averse and more risk averse than a well-paid executive of a large corporation. Someone who rarely faces gambles is more likely to be risk averse and more risk averse for a specific gamble than someone who routinely gambles on his or her own account.

Second, the theory of expected utility maximization is not meant to be a *literal* description of how individuals' make decisions. Like many theories in economics, it is an *as-if model.* Individuals facing risky decisions do not actually compute expected utilities and choose according to the results of those calculations, like we did here. Rather, they act *as if* they did these sorts of calculations—and, because of that, the theory can give us useful predictions and help understand how much people will pay, in real cash, to insure against a potential loss, to hedge a gamble, or to acquire a risky investment.

The natural question to proceed to now is, How does this theory help us understand how real markets work? We'll tackle this question by working through an important application: Investment pools and securitization. This will illustrate some fundamental lessons about how markets deal with risk.

Problem to Prepare for Class

Investment Pools and Securitization

This is a discussion with calculations for you to do along the way. Please be prepared to discuss your answers in class. As usual, you do not need to turn in your work on class-session problems, and you may work with others if you wish. Note: Although the calculations are not complex, ***please use a spreadsheet for all calculations to avoid rounding error.***

Jan MBA has identified a risky venture opportunity that requires an up-front capital investment of $60,000 to get going. The downside is that there's a 75% chance the venture will be unsuccessful, in which case the entire investment of $60,000 will be lost. The upside is that there's a 25% chance the venture will be successful. In that case the venture will pay out gross proceeds of $400,000, giving Jan her initial investment of $60,000 back plus a handsome capital gain of $340,000.

A. *A Single Risk-Averse Investor*

Jan MBA's initial wealth (W) is $100,000. She knows that, if she decides to invest $60,000 in the venture, her final wealth will be either $40,000 or $440,000, with 75/25 chances, respectively. So the EMV of her wealth *if* she decides to invest in the venture is

$$\text{EMV} = \$40{,}000 \times .75 + \$440{,}000 \times .25 = \$140{,}000.$$

This is $40k better than her initial wealth. But Jan is risk averse: She has utility function $U = W^{0.5}$. This is close to being linear for small risks, but bends down slowly as it rises with wealth. Let's suppose, for the moment, that investing $60,000 in this venture is Jan's only opportunity to employ her wealth.

(a) What is Jan's expected utility if she invests $60,000 in the venture?

(b) What is Jan's *certainty equivalent wealth* for this investment opportunity? *N.B.*, this is the amount of money-for-sure that gives Jan the same utility as in part (a).

Jan MBA will be better off investing in the venture if her certainty equivalent wealth beats her alternative opportunity, which is sitting on her initial wealth of $100,000.

Key Point 1. In practice, certainty equivalents and risk premia are a much more useful way to communicate about risk tolerance than is expected utility. (This much is probably obvious). And you can work with either in the end. If a particular gamble has a given CE, you compare this CE to the wealth level under your next best alternative and invest if your CE is the higher of the two. The *risk premium* tells us how much this investor's expected wealth (under the gamble) exceeds the money needed for certain to be indifferent to the risk. Or, in somewhat more intuitive terms, the RP is how much expected wealth she'd forgo to avoid this risk. A less risk-averse investor would enjoy a smaller risk premium. This is one (often the main) reason why hedge funds and large investment pools like to have only wealthy investors—the risk premium the fund must pay to attract capital is lower, since the risk premium falls as wealth rises.

B. *A Single, Less Risk-Averse Investor*

Jan MBA knows another, wealthier investor, Professor C. Gall. This investor has the same utility function as Jan, or $U = W^{0.5}$. However, Professor C. Gall has greater initial wealth of $1,000,000. As before, if Professor C. Gall decides to invest $60,000 in the venture he will face the same possible venture outcomes and probabilities. This gives him final wealth levels of either $940,000 or $1,340,000 with 75/25 chances, respectively.

(a) What is Professor C. Gall's expected utility if he invests $60,000 in the venture, and what is his certainty equivalent? What will he do?

Jan thinks about his. Even if she wasn't willing to invest in the venture with her own money, the fact that someone else would invest in it suggests she is giving away a valuable item. Perhaps Jan ought to *sell* the opportunity to invest in her venture.

Instead of just telling him about the venture, Jan decides to *sell* the opportunity to Professor C. Gall for a price P (where P > $60,000). Jan will take $60,000 of the amount she receives up-front from Professor C. Gall and invest it in the venture, keeping the remaining amount (or P – $60,000) for herself. Jan tells Prof. C. Gall that, in consideration of his investment of $P, Professor C. Gall will get nothing in return if the venture fails but will receive the entire gross proceeds of the venture if it is successful (i.e., $400,000). Professor C. Gall knows the chances of success are only 25%.

(b) What is the *maximum* price $P that Professor C. Gall would be willing to pay to Jan for the opportunity to invest in the venture?

Your answer should have the form of: He would pay up to ($60,000 plus *something*). Note that Jan keeps the *something* part for herself, yet has taken on no risk here at all. Not bad for an investment opportunity she was originally planning to decline.

C. *Securitization and Risk Sharing with Many Investors*

Suppose now that no investor like our former wealthy investor in Part B can be readily found, but there are lots of investors with lower wealth levels like Jan MBA. Does this mean that no one will provide the $60,000 in capital to get this venture off the ground? **No.** In fact, most financing arrangements—whether they are issuing stock on a public exchange, or raising venture capital, or whatever—are designed to deal with exactly this circumstance: The risk is too big for any single investor to be willing to take it on alone.

The simplest solution to this is called *securitization.* This is a fancy-schmancy term for chopping up the investment risk into small pieces, and selling each piece to a different investor. It works because *every person is less risk averse when the stakes are smaller.*

Let's continue the example. Suppose we find 1,000 people, each one exactly like Jan MBA (i.e., each with a utility function of $U = W^{0.5}$ and each with initial wealth of $100,000). Even though none of them would individually put up $60,000 to fund the venture, suppose we instead form an investment pool into which each person invests only \$60. Jointly, this would raise the \$60,000 in capital needed to get the venture going. Since all 1,000 participants are funding the venture equally in this example, let's assume the pool allocates the total investment gain or loss among them equally. Thus, if the investment fails, each member loses his or her principal (i.e., loses \$60), while if the investment is a success each member gets his or her \$60 back *plus* $1/1000^{th}$ of the total investment gain, for a net gain of \$340 each.

Once the big investment (of \$60,000) is securitized into little pieces requiring an investment of only \$60 per person, the expected monetary value of the investment to an individual member is $-\$60 \times 0.75 + \$340 \times 0.25 = \$40$. The expected monetary value of an individual pool member's wealth is then $\$99{,}940 \times 0.75 + \$100{,}340 \times 0.25 = \$100{,}040$. Will an individual join? As we know, the answer does not depend on the expected value of wealth, but on the expected utility of wealth.

(a) Find the expected utility and certainty equivalent wealth for an investor who puts \$60 into this pool, assuming $U = W^{.5}$ and \$100,000 in initial wealth.

In case the profound consequences of this last calculation went by fast amidst all these numbers, let's make use of it in the way a shrewd, economics-savvy MBA like Jan would. Specifically, suppose Jan decides to sell *shares* in her venture. Each share entitles the buyer to $1/1000^{th}$ of the venture's gross proceeds, or \$400, if the venture is successful. However, the buyer pays the price-per-share to Jan up front, and the share price is (of course) non-refundable if the venture fails.

(b) What is the *maximum* that an investor with the same preferences as Jan (that is, with initial wealth \$100,000 and utility $U = W^{.5}$) would be willing to pay for *one share* in Jan's venture?

(c) Assuming she sells all 1,000 shares, puts \$60,000 into the venture, and thus faces no remaining risk, how much money does she make?

Note that, if you did this right, your answer should be getting pretty close to the EMV of the venture that we calculated back in part A. Moreover, the amount she makes here is better than the amount she could get from Prof. C. Gall in part B, even though he was a lot wealthier. *Think about that.*

Key Point 2. This risk-sharing lesson is the economic basis for all risk-spreading entities, including equity (stock) markets, hedge funds, investment pools, syndicates, many joint ventures, and so on. You can make money by helping firms (new or existing) take their big risks and big capital requirements, chopping them up into tiny pieces, and re-selling smaller pieces of this risk to thousands of individual (or institutional) investors. The analysis in this example is the fundamental economic rationale for why firms go public, and why there is a business of equity underwriting. Now you know.

About Risk Premia

Let's restate all this in terms of risk premia. Consider first consider the risk premium for Professor C. Gall, in question (a) of part B. If you did the math right there, this works out to RP = EMV – CE = $6,630.36. The interpretation of this risk premium is that Professor C. Gall would forego $6,630.36 in expected gain before he would be willing to take on the venture's risk (that is, paying nothing for the opportunity to invest). Or, put another way, the risk premium that Professor C. Gall requires in order to put up the $60,000 in capital to get this venture going is $6,630.36.

By contrast, consider the risk premium when the investment is *securitized* among 1,000 investors. An investor that puts $60 into the pool has a risk premium of RP = EMV – CE, which works out to 7.49 cents per investor. Summed over all 1000 investors, the aggregate RP these investors require is $74.90. The interpretation of this aggregate risk premium is that, by holding one $1/1000^{th}$ share each, these investors would be willing to forego a total of $74.90 in expected gain before being willing to take on the risk. Or, put another way, the risk premium that our 1,000 investors require in order to put up the $60,000 in capital to get this venture going is a paltry $74.90.

Key Point 3. So, if you are trying to raise capital from the community of investors, how do you minimize the total risk premium you must offer to raise a large amount of capital? Chop up the risk into small pieces and sell shares to lots of people.

D. *Partial Ownership: When You Know More than the Market*

Read this to appreciate Key Point 4 at the end. Note: the calculations in this part are optional, and you do not need to prepare them for class. However, if you choose to do them, they will help you better understand how markets work.

Suppose that, in contrast to our analysis up to this point, Jan is convinced that the venture will pay out gross proceeds of $400,000 with probability 1/3, and fail with probability 2/3. But everyone else in the world thinks the two probabilities are .25 and .75, as before. Because of the latter fact, the most that Jan can sell her $1/1000^{th}$ shares in her venture is the price per share you obtained in part C(b).

How can she profit from her knowledge that the venture has a higher chance of success than the market? If you go back and evaluate Jan's expected utility and CE if she invests in the venture with her own money (like in Part A) using a probability of success of 1/3, her CE wealth for the venture is $125,628.89. That's over $25k better than if she sits on her $100,000 in initial wealth and does nothing, so she would be *willing* to take it on herself. [*Note: If you set up a spreadsheet to do all the calculations so far, this should be easy to verify by changing the assumed probabilities at the start. As I said, use a spreadsheet!*]

Still, the CE of the venture using her knowledge of the higher success probability (that is, $125,628.89 – $100,000 = $25,628.89) is less than the money-for-sure she can get from selling shares in the venture to a more pessimistic market, as we found in part C. So it looks like selling all the shares in the venture, for the price you found in part C(b), is the best thing for her to do. Perhaps there is no way to exploit her private information about the higher chance of success after all.

Or… maybe there is. Since Jan has a CE for the venture that is positive when the chance of success is 1/3, *maybe* she could do better if she kept some of the shares for herself. Suppose Jan divides the venture into 1/1000th shares, as before, but now is considering keeping some number (necessarily between 0 and 1000) of the shares for herself. Since outside investors assess the chance that the venture succeeds as being only 0.25, we know the price that Jan can get for each share she sells to an outside investor the price you obtained in Part C(b).

(a) Jan can choose any number n (between 0 and 1,000) of shares to retain for herself. Each share pays $400 if the venture succeeds, and zero if it fails. How many shares should Jan retain for herself? *Remember:* Jan knows the chance the venture succeeds is 1/3, whereas the rest of the world believes it is 0.25.

(b) What is her CE wealth *including* her profit (risk-free!) from selling the other (1000 – n) shares?

And to think, she was originally going to turn down the gamble!

Key Point 4. Although there are several reasons why entrepreneurs retain substantial shares of their firms when firms secure outside financing, markets tend to view a firm's prospects more favorably if the entrepreneur (at a venture capital financing stage) or the firm's professional management (at an IPO stage) retains a large share of the firm. The reasoning is exactly the same as we've seen here, but in reverse: The market interprets a large ownership share of the insiders as a signal that these insiders view the firms chances of success as high.

The other reason that entrepreneurs and managers retain large shares of firms after securing outside financing is *incentives.* And incentives, in this context, will be our topic for class session 10.

Managerial Implications

Although the mechanics of the problems in this note will prove to be pretty straightforward once you get the hang of it, the conceptual points in this note are quite deep.

1. The major economic rationale for the existence of equity, insurance, and futures markets is this: By taking a risk and breaking it into pieces, with the pieces shared among many individuals—in other words, *securitizing* the risk—risk aversion on the part of individual (or even institutional) investors can be defeated. This lowers the cost of capital throughout the economy.

2. Clearly, one can make money by helping firms (new or existing) take their big risks and big capital requirements, chopping them up into tiny pieces, and reselling smaller pieces of this risk to thousands of individual (or institutional) investors. The reason for this is that people become close to risk neutral as the stakes of their investment become small, and therefore will accept a smaller expected wealth gain and still be willing to provide investment capital.

3. In insurance markets, efficiency requires the ability to price discriminate among individuals with differing levels of risk. When individuals have better information about their likelihood of incurring a loss than an insurer, offering a menu of partial insurance contracts allow markets to function. *Without price discrimination it may be unprofitable for anyone to provide insurance.*

Appendix: What's My Risk Aversion?

Here is a simple exercise to measure your own degree of risk aversion. Imagine you are choosing between a job with a guaranteed summer salary, and another job which has a salary and chance at an end-of-summer bonus. In this second job, the salary is $20,000 for the summer. The bonus is $10,000 if your division hits its sales target, and $0 if it does not. After interviews with people at the second job, you estimate that if you join the firm there is a 50/50 chance the division will meet its sales target. Thus your expected salary is:

$$E(\text{Salary}) = (.5)\cdot 20{,}000 + (.5)\cdot 30{,}000 = \$25{,}000.$$

You are now negotiating with the first firm, which offers only a fixed salary for the summer. In anticipation of your meeting with your potential managing partner, write down the salary that you think will be just enough to convince you to take this "sure salary" job rather than the "salary and possible bonus" job. Or, to put matters slightly differently, what is your "cutoff" salary in the "sure salary" job—that is, the salary where, if they offer $1 less, you decide you would rather take the bonus job? It is at this salary that you are just "indifferent" between the safe and risky compensation schemes.

My "indifference" salary is: ______________________________.

Given your answer, here is how to now compute your measure of risk aversion. Assume your utility function in wealth is:

$$U = A + (1/a) \times W^a,$$

where A is a constant (the value of A does not affect the curvature of U, so it is irrelevant for measuring risk aversion).[1] Using this expression for U(W), set your utility of your sure salary S just equal to the utility you would get from the bonus plan job:

$$U(S) = .5\ U(W = 20{,}000) + .5\ U(W = 30000).$$

Since you said that S for sure is just as good as the risky bonus, you were defining S to satisfy that expression above. (When you do the math you will notice that the A's cancel from both sides of the equality and so will the $(1/a)$'s. Remember we said all that really matters when making choices is the exponent 'a' attached to wealth.) You will obtain the expression:

$$(S)^a = .5 \cdot (20000)^a + .5 \cdot (30000)^a .$$

Substitute in your S and solve for a. Mathematically, if a gets very close to 0, then U = A + log(W) should be used. Typically a will be less than 0. To solve for ***your a*** use the formula above and a spreadsheet, say beginning with $a = 0$ (or U = log(W)) and then try a = -1, -2, -3, ... – 10, etc. Here is what you will get, approximately. If you said:

- Salary = 21,400 or lower, then $a = -10$ or a bit more negative.
- Salary = 22,400 or thereabouts, then $a = -5$.
- Salary = 23,100 or thereabouts, then $a = -3$.
- Salary = 24,000 or thereabouts, then $a = -1$.
- Salary = 24,500 then $a = 0$.
- Salary = 25,000 then $a = +1$. You are risk neutral.

What do we know about "real" values of a's in markets? It is a matter of some controversy in finance. Analyses of investor portfolios imply values of $-1 > a > -3$. Trying to unravel the equity premium puzzle, however, finance research suggests a may be as small as –6 or less. That implies investors are very risk averse, and is why they need such a large premium to invest in equities.

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[1] If you wish, draw this utility function. It rises steadily from $W \cong 0$ towards A. The marginal utility of wealth is given by $\partial U/\partial W = W^{(a-1)} > 0$, and marginal utility declines as wealth increases for all values of $a < 1$. That is, $\partial^2 U/\partial W^2 = (a-1)W^{(a-2)} < 0$.