

Discounting and Optimism Equivalences

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Abstract

Both discounting and optimism can induce impatient behavior in the present. This paper derives equivalences between classes of discount functions and classes of (possibly erroneous) sets of beliefs in a standard intertemporal choice context. We find that strong assumptions about beliefs are generically necessary for identification of the discount function.

1 Introduction

Given a choice set, beliefs and preferences define an individual decision problem. This paper studies equivalences between classes of beliefs and classes of preferences in the context of intertemporal choice decision problems. Qualitatively, one might expect non-identification of particular classes of beliefs and preferences to arise in an intertemporal choice context in the following way. As beliefs about the future state of the world (i.e., future income) become more optimistic, higher consumption levels in the present are induced since consumption in all periods is a normal good. Moreover, overoptimistic consumers will implement downward-sloping consumption paths, relative to what they have planned, as the reality of less fortunate future states sets in. Higher discounting also, of course, causes the slope of the consumption path to fall. Our main result is that for most discount functions, including all of the most commonly analyzed discount functions, changes in the parameters of the discount function that decrease patience have effects on consumption paths that can be *exactly* offset by making beliefs more optimistic.

Economists usually assume beliefs are tied down by information and feedback from the market, Bayesian updating, and strong incentives to be correct. In the case of future income, overoptimistic beliefs could persist for a variety of reasons.¹ First, moral hazard may limit informative market offers of credit

¹We say a consumer is “overoptimistic” if the probability distribution she has for future income first-order stochastically dominates the true distribution. Other forms of overoptimism are also possible. For risk averse consumers, we could demonstrate equivalences analogous to those proven below, by producing beliefs about future income that second-order stochastically dominate the true distribution. “Naivete” (Akerlof 1991, O’Donoghue and

linked to future income.² Second, individual heterogeneity in income is wide, implying many signals may be necessary to gain precision (Güvener 2007). Third, more generally, overoptimism might be evolutionarily adaptive, counteracting (small-stakes) risk aversion. Fourth, Brunnermeier and Parker (2005) conjecture that consumers may prefer to believe they will experience favorable outcomes, and willingly accept the consequences of the poorer decisions required to accord with their overoptimistic beliefs. Fifth, Malmendier and Tate (2005) illustrates a winner’s curse mechanism that can sustain overoptimism about chosen tasks: if there is heterogeneity in beliefs about the return to an action, the individuals with the highest estimates of returns will be the only ones to undertake it. Finally, firms in competitive markets generally may not have incentives to disabuse consumers of overoptimism (Gabaix and Laibson 2006, Ellison 2005). These reasons complement empirical findings in the psychology literature that overoptimism is common (Weinstein 1980, e.g.).

We remain agnostic about possible causes of overoptimism, and focus on identification relationships between overoptimism and discounting.³ Section 2 presents an illustrative example. Sections 3 and 4 present our main results and discuss generalizations, before we conclude in Section 5.

2 Illustrative Example

To illustrate the idea, consider the following extremely simple two period consumption problem. For $t \in \{1, 2\}$, the consumer chooses consumption c_t . Preferences, with discount factor δ , are given by

$$U_1(c_1, c_2) = u(c_1) + \delta u(c_2).$$

Let R be the (fixed, known) gross interest rate, and denote income by $y_t \geq 0$. Then the budget constraint is

$$y_1 + y_2/R \geq c_1 + c_2/R,$$

assuming that there are no liquidity constraints.

Suppose the consumer *expects*, incorrectly, that income next period will be y_2^E , which could be different from y_2 . Denote expected period 2 consumption by c_2^E . The consumer (incorrectly) believes⁴ the budget constraint to be

$$y_1 + y_2^E/R \geq c_1 + c_2^E/R.$$

Rabin 1999b, O’Donoghue and Rabin 1999a, O’Donoghue and Rabin 2001) can be viewed as overoptimism about one’s own future preferences, when a consumer is dynamically inconsistent. “Overconfidence” Malmendier and Tate (2005) could be overoptimism about one’s returns to effort.

²For evidence that liquidity constraints bind for many consumers, see Mankiw and Zeldes (1991) and Hubbard, Skinner and Zeldes (1995).

³This paper also contributes to the extensive literature on identification– and limits thereto– using consumption data (for one example, see Browning (1997)).

⁴“Expect” and “believe” are used differently here than usual: for simplicity we assume consumers think they know the future *perfectly*, but somehow what they know is wrong.

The solution is obtained via the familiar Euler Equation:

$$u'(c_1) = \delta R u'(c_2^E)$$

In equilibrium the consumer leaves no resources unconsumed in period 2:

$$\begin{aligned} c_2^E &= y_2^E + R(y_1 - c_1) \\ c_2 &= y_2 + R(y_1 - c_1) \end{aligned}$$

Plugging into the Euler Equation we obtain,

$$u'(c_1) = \delta R u'(y_2^E + R(y_1 - c_1)).$$

Thus, with an invertible $u'(\cdot)$, it is obvious that c_1 can be increased smoothly either by reducing δ or by raising y_2^E . Observing $\{y_1, c_1, R\}$, we can't distinguish whether an individual is impatient (ie, has low δ) or optimistic (ie, has high y_2^E). Impatient and overoptimistic consumers will even be liquidity constrained for exactly the same range of y_1 values.

3 Exponential Infinite-Horizon Problem

Next we examine an infinite-horizon version of the model. We retain considerable simplicity, since additional degrees of freedom will confound the relationship between discounting and overoptimism. (Too many degrees of freedom could prevent identification of either alone, making it unsurprising if the two could not be separated.)

Assume perfect (possibly erroneous) certainty and perfect markets, and also CRRA preferences. Total utility,

$$U_t \left(\{c_\tau\}_{\tau=t}^T \right) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau),$$

is maximized subject to a budget constraint. At time t , before the period's consumption is chosen, denote the NPV of total remaining lifetime wealth by

$$w_t = \sum_{i=-t}^{\infty} \frac{y_{t+i}}{R^i} - \sum_{i=-t}^{-1} \frac{c_{t+i}}{R^i}.$$

Even with perfect markets, we must always have $w_t \geq 0$. For now, assume this condition holds; we will return to discuss it below.

Let y_{t_1, t_2}^E be the income self t_1 believes will be received at time t_2 . Assume consumers have accurate beliefs about current and past income, so $y_{t_1, t_2}^E = y_{t_2}$ $\forall t_1 \geq t_2$. The overoptimistic consumer at time t perceives the NPV of total remaining lifetime wealth to be:

$$w_t^E = \sum_{i=-t}^0 \frac{y_{t+i}}{R^i} + \sum_{i=1}^{\infty} \frac{y_{t, t+i}^E}{R^i} - \sum_{i=-t}^{-1} \frac{c_{t+i}}{R^i}.$$

Assume beliefs about future income are believed to be common knowledge, ie, assume that $y_{t_1, t_3}^E = y_{t_2, t_3}^E \forall t_1, t_2 < t_3$, and higher order agreement on income at t_3 also prevails for selves prior to t_3 , so we can write $y_{t_3}^E$, with a single subscript.

We seek an equilibrium in Markov strategies. Since preferences are CRRA we can normalize by a wealth measure, implying the equilibrium can be written thus:

$$c_t = \lambda^E w_t^E \forall t. \quad (1)$$

We take the Markov perfection requirement to mean, additionally, that self t believes future selves will play the same Markov equilibrium, even though self t has different beliefs about w_{t+i}^E than future selves will because future selves will have observed additional income realizations. Thus our form of Markov perfection also requires

$$c_{t,t+1}^E = \lambda^E R (w_t^E - c_t).$$

Algebraic manipulation of the Euler Equation that defines optimality at time t ,

$$u'(c_t) = \delta R u'(c_{t,t+1}^E),$$

implies

$$\lambda^E = 1 - (\delta R^{1-\rho})^{1/\rho}.$$

The expression for equilibrium consumption choices,

$$c_t = \left(1 - (\delta R^{1-\rho})^{1/\rho}\right) w_t^E,$$

suggests that there may be some equivalence between δ and w_t^E .

Conversion into consumption growth rates illuminates further. Actual consumption at $t+1$ is given by

$$c_{t+1} = \lambda^E w_{t+1}^E,$$

and

$$w_{t+1}^E = R \left(1 - \lambda^E\right) w_t^E - (y_{t+1}^E - y_{t+1}),$$

where the first term represents the effect of consumption at time t and interest earnings, and the second term represents the effect of learning the true value of y_{t+1} . Combining with the expression for c_t we obtain,

$$\frac{c_{t+1}}{c_t} = \frac{w_{t+1}^E}{w_t^E} = R \left(1 - \lambda^E\right) - \frac{(y_{t+1}^E - y_{t+1})}{w_t^E} \quad (2)$$

$$= (\delta R)^{1/\rho} - \frac{(y_{t+1}^E - y_{t+1})}{w_t^E} \quad (3)$$

Given R and ρ , this equation implies that δ is indistinguishable from overoptimism if and only if the amount of overoptimism about tomorrow's income is a constant share of what is believed to be (remaining) lifetime wealth. Thus we have shown:

Proposition 1 For any constant consumption growth rate, given R and ρ , (i) for any exponential discount factor, $\exists \{y_t^E\}$ such that the specified consumption growth rate prevails in a Markov Perfect Equilibrium; and (ii) if $\exists \phi$ such that $\phi = \frac{(y_{t+1}^E - y_{t+1})}{w_t^E} \forall t$, there exists an exponential discount factor δ such that the specified consumption growth rate prevails in a Markov Perfect Equilibrium.

Since beliefs about lifetime wealth are falling for overoptimistic consumers (as they receive actual income disappointments), the amount of overoptimism about income must also be falling period-by-period. In fact, the requirement that $\phi = \frac{(y_{t+1}^E - y_{t+1})}{w_t^E} \forall t$ implies

$$\begin{aligned} w_t^E &= \sum_{i=-t}^0 \frac{y_{t+i}}{R^i} + \sum_{i=1}^{\infty} \frac{y_{t+i}^E}{R^i} - \sum_{i=-t}^{-1} \frac{c_{t+i}}{R^i} \\ &= w_t + \sum_{i=1}^{\infty} \frac{y_{t+i}^E - y_{t+i}}{R^i} \\ &= w_t + \sum_{i=1}^{\infty} \frac{\phi w_{t+i-1}^E}{R^i}. \end{aligned}$$

Also,

$$\begin{aligned} w_{t+1}^E &= (\delta R)^{1/\rho} w_t^E - (y_{t+1}^E - y_{t+1}) \Rightarrow \\ \frac{w_{t+1}^E}{w_t^E} &= (\delta R)^{1/\rho} - \frac{(y_{t+1}^E - y_{t+1})}{w_t^E} \Rightarrow \\ \frac{w_{t+1}^E}{w_t^E} &= (\delta R)^{1/\rho} - \phi \Rightarrow \\ \frac{w_{t+i}^E}{w_t^E} &= [(\delta R)^{1/\rho} - \phi]^i, \quad \forall t, i, \quad \text{so} \\ w_t^E &= w_t + w_t^E \sum_{i=1}^{\infty} \frac{\phi [(\delta R)^{1/\rho} - \phi]^{i-1}}{R^i} \\ &= w_t + w_t^E \frac{\phi}{R} \sum_{i=0}^{\infty} \left[\frac{(\delta R)^{1/\rho} - \phi}{R} \right]^i \\ &= w_t + w_t^E \frac{\phi}{R - (\delta R)^{1/\rho} - \phi}, \end{aligned}$$

implying

$$w_t^E = \frac{w_t (R - (\delta R)^{1/\rho} - \phi)}{(R - (\delta R)^{1/\rho} - 2\phi)}.$$

Thus $w_t^E > w_t$, but $\lim_{t \rightarrow \infty} w_t^E = w_t$.⁵ We conclude that there is a *unique* way to manipulate overoptimism to offset a given change in δ ; and changes to the structure of overoptimism generically (in the space of sequences $\{y_t^E\}_{t=0}^\infty$) cannot be offset by changes in δ .

4 Generalizations

Section 3 made strong assumptions about the instantaneous utility function, knowledge (by the consumer and the econometrician) of the interest rate and consumption choices, and the distribution of income in each period. In this section we discuss the effect of relaxing these and other assumptions.

4.1 The Instantaneous Utility Function

Retaining the assumption that preferences are CRRA, but assuming ρ is unknown would suggest (cf Equation 3) that risk aversion is also not separately identifiable from overoptimism. However, a variety of independent approaches are available to estimate ρ . Moreover, separation between overoptimism and ρ would be less complete, because changes in ρ can never affect the *sign* of the slope of the consumption path, only the steepness of the path.

If we relax the CRRA assumption, the convenient (normalized) representations of the equilibrium we used above are unavailable. The basic non-identification result continues to hold, but it will no longer be true that the amount of overoptimism about tomorrow's income is a constant share of what is believed to be (remaining) lifetime wealth.

4.2 Interest Rates, and Measurement

If interest rates vary deterministically over time, so that R_{t+1} prevails from t to $t+1$, trivial modifications to the main derivation above imply an analogous non-identification result if $\phi_t = \frac{(y_{t+1}^E - y_{t+1})}{w_t^E R_{t+1}^{1/\rho}}$ is constant.

Separately, interest rates, consumption, and income might be measured with error. Then there may not exist a discount factor that exactly rationalizes the measured consumption path. Regardless, any consumption path that can be implied by variations in the discount factor can also be implied by variations in overoptimism as shown.

4.3 Uncertainty about Income, and Learning

Contrary to the assumption in the previous Section, the actual and believed income distributions may not be degenerate. Adding generality on this point, has little effect on our result. Given an instantaneous utility function $u(\cdot)$, a

⁵Per the comment at the top, note that $(R - (\delta R)^{1/\rho} - 2\phi) > 0$ is necessary and sufficient to have $w_t > 0$ for all t .

fixed interest rate R , and the true joint distribution of income $g(\{y_t\}_{t=0}^\infty)$, for any consumption path $\{c_t\}_{t=0}^\infty$ and discount function $f(t)$ that rationalizes that consumption path, there exists a set of beliefs about the joint distribution of income $g^E(\{y_t^E\}_{t=0}^\infty)$ that rationalizes the same consumption path for $f(t) = 1$.

Secondly, despite the reasons presented in the Introduction that overoptimistic beliefs about income might persist, usually economists assume beliefs are tied down by Bayesian updating on available information. Workers might begin their careers with overoptimistic beliefs (ie, at time 0 the prior joint distribution of income could have partials for every period's income that FOSD the actual per-period distributions) but become more accurate over time. We certainly find it more plausible that consumers hold beliefs about the distribution of possible income realizations that share some of the support of the true distribution. Guvenen (2007) suggests the way to the next step of studying equivalences between classes of discount functions and classes of *initially* overoptimistic beliefs, when the evolution of beliefs over time is constrained by Bayesian updating.

4.4 Other Discount Functions

The derivation of the equivalence between exponential discounting and overoptimism in Section 3 can be generalized easily for other discount functions. Equation 2 relies on the existence of some λ^E such that the equilibrium is characterized by

$$c_t = \lambda^E w_t^E \quad \forall t.$$

In the exponential case an explicit representation for λ^E is available. However, Laibson (1997) shows that sophisticated quasi-hyperbolic discounting also results in a unique Markov Perfect Equilibrium in the infinite horizon, characterized implicitly by the solution to the Hyperbolic Euler Equation:

$$\begin{aligned} \lambda^E &= 1 - \left\{ R^{1-\rho} \delta \left[\beta \lambda^E + (1 - \lambda^E) \right] \right\}^{1/\rho} \\ c_t &= \lambda^E w_t^E \quad \forall t \end{aligned} \quad (4)$$

MPE's are also available for other generalized discount functions.⁶

4.5 The Finite Horizon, and Converse Counterexamples

So far this paper has primarily considered Markov Perfect Equilibria in infinite-horizon settings. Over finite horizons, backward induction results in unique subgame perfect equilibria for any discount function. Non-exponential discount functions can produce non-monotonic equilibrium consumption functions, implying that non-monotonic consumption paths are possible. It should be possible to either produce a set of beliefs that results in a consumption path that cannot

⁶Like in most infinite horizon games, weaker refinements (including subgame perfection) result in multiplicity of equilibria. Krusell and Anthony A. Smith (2003) study the range of equilibria in the quasi-hyperbolic case. It is unknown whether set of possibly-overoptimistic beliefs could be constructed to replicate consumption paths in all of these equilibria.

be generated as an SPE for any discount function; or to show how to construct a discount function that replicates the consumption path for any set of beliefs.

4.6 Other Types of Overoptimism

Following on Strotz (1955)'s analysis of "consistent planning," theoretical insight into naive hyperbolic discounting was provided by Akerlof (1991) and O'Donoghue and Rabin (1999a, 1999b, 2001). Tobacman (2007) indicates theoretically that the behavior of sophisticates and naifs may differ little in standard consumption models for typical parametrizations, consistent with Angeletos, Laibson, Repetto, Tobacman and Weinberg (2001)'s simulation evidence. In addition, in some applications including those studied by DellaVigna and Malmendier (2004, 2006), overoptimism about the future state of the world is indistinguishable from overoptimism about future preferences. Fang and Silverman (2004b), Shui and Ausubel (2004), and Skiba and Tobacman (2006) estimate models of naive hyperbolic discounting, using data on the duration of welfare spells, on credit card take-up and use, and on payday loan use and default, respectively. In those contexts, the opportunity to procrastinate distinguishes discounting from overoptimism.

5 Conclusion

This paper's non-identification results have three main implications. First, when seeking to identify discount functions, data on consumption and interest rates will not suffice. A number of authors have used more specialized data (eg, on illiquidity) to pin down discount function parameters (Fang and Silverman 2004a, Ashraf, Karlan and Yin 2006, Laibson, Repetto and Tobacman 2007, Skiba and Tobacman 2006, for example), but even they have generally not allowed beliefs about the state of the world to vary simultaneously.

Second, direct measurement of beliefs should complement data on consumption and interest rates, and thereby help to cut the identification knot. Ameriks, Caplin, Laufer and Nieuwerburgh (2007) and Stango and Zinman (2007) pursue this for practical reasons, and Caplin (2007) explains how to ensure the approach remains disciplined.

Third, modelling innovations often modify either the structure of preferences or the structure of beliefs. Often it is unclear where to innovate. This paper indicates, at least in a specific domain, that the choice may matter little.

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